Abbot Hall Kendal 1797

THE COMPLETE MEASURER: OR, THE *Whole Art of Measuring.*

IN TWO PARTS.

The First PART teaching
DECIMAL ARITHMETICK, with the *Extraction* of the SQUARE and CUBE ROOTS:

And also the *Multiplication of Feet and Inches*, commonly called CROSS MULTIPLICATION.

The Second PART teaching to
Measure all Sorts of SUPERFICIES and SOLIDS, by *Decimals*; by *Cross-Multiplication*, and by *Scale and Compasses*: Also the Works of several Artificers, relating to *Building*; and the Measuring of *Board* and *Timber*. Shewing the common Errors.

And some *Practical* QUESTIONS.

The *Eleventh* EDITION. To which is added, *An* APPENDIX. 1. *Of* Gauging. 2. *Of* Land-Measuring.

Very useful for all Tradesmen; especially Carpenters, Bricklayers, Plasterers, Painters, Joiners, Glasiers, Masons, &c.

By WILLIAM HAWNEY, *Philomath.*

Recommended by the Rev. Dr. John Harris, *F. R. S.*

LONDON:

Printed for J. KNAPTON, H. WOODFALL, J. RIVINGTON, R. BALDWIN, L. HAWES, W. CLARKE and R. COLLINS, J. RICHARDSON, G. KEITH, S. CROWDER and Co. T. LONGMAN, B. LAW and Co. R. WARE, and J. HINXMAN. 1763.

I Have peruſed this BOOK, and recommend it to the Publick as a very uſeful One.

J. Harris, *D. D.*

Lately publiſhed, the 2d Edition of

The Doctrine of Plain and Spherical TRIGONOMETRY; with its Uſe in various Branches of the Mathematicks.

By *W. Hawney*, Author of this Book.

Alſo The Eighth Edition of

Pardie's Short but yet Plain Elements of GEOMETRY; Shewing by a brief and eaſy Method how moſt of what is Neceſſary and Uſeful in this Science, may be underſtood. Tranſlated by the above-mentioned Dr. *Harris*, and referred to in the Preface of this Book.

THE

PREFACE.

HAVING perused several Books, concerning the Mensuration of Superficies and Solids, and the Works of Artificers relating to Building; but not finding any one Book so perfect, as to give any tolerable Satisfaction to a Learner; and I having practised and taught Measuring for several Years, and thereby gained Experience and Knowledge in that Art, having learned some Things from one Author, and some Things from another, I began to think of digesting my Thoughts into some such Method as might give a Learner full Satisfaction, without being at the Charge of buying so many Books; and being importuned thereunto by some Friends, I fell to work, and at last brought them to that Perfection you here find in the following Work.

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1. As to the *Decimal Arithmetick*, I have been as brief as the Matter would well bear, to make it plain.

2. As to the Multiplying of Feet and Inches, commonly called *Cross-Multiplication*, my Method differs from that which is usually taught in other Authors, as being (I think) much shorter and plainer.

3. In measuring of Superficies and Solids, I have given the Demonstration of the Rules, which I thought might be very acceptable to the Ingenious; for, indeed, I always look upon the Writing of a Rule without a Demonstration (in any Part of the Mathematicks) to be but lame and defective; and for want of knowing the Reason of the Rule, a Learner may commit great Errors; besides, when a Learner knows the Reason of the Rules, he may retain them better in his Memory. The Rule for measuring a Prismoid and Cylindroid, I had out of Mr. *Everard's Art of Gauging*; but the Reason he does not shew, neither have I found it in any other Author; but that the Method is true, I have endeavoured to make plain.

The Demonstration of the Rules for finding the Area of an Ellipsis and Parabola; also the Demonstration of the Rules for finding the solid Content of the Frustum of a Cone and Pyramid, the Solidity of a Globe of a Spheroid, a Parabolic Conoid, and of a Parabolic Spindle, and their Frustums, I had from the ingenious Mr. *Ward's Young Mathematician's Guide*; where the curious and ingenious Reader may see many other Demonstrations algebraically performed. I have also demonstrated the Rule for finding the Solidity of a Globe out of *Pardie's. Elements of Geometry* (Book the 5th, Art. the 33d) published in *English* with many Additions,

Additions, by the Reverend Dr. *Harris*, F. R. S. and the ſame is alſo done out of *Sturmius's Matheſis Enucleata*; ſo that the ingenious Reader may uſe which of thoſe Ways he likes beſt.

The Scale ſuppoſed to be uſed in all the Operations, is the Line of Numbers, commonly called *Gunter's Line*, which is upon the ordinary Two-Feet or Eighteen-Inch Rules, commonly uſed by the Carpenters, Maſons, *&c.* becauſe I thought it needleſs, as well as impertinent, to write the Uſe of Sliding-Rules, or any other particular Scales, they being ſufficiently treated of by ſeveral Authors; *viz.* by the above-named Mr. *Everard*, in his *Art of Gauging* above-mentioned, where you have the Uſe of a Sliding-Rule in Arithmetick, Geometry, in Meaſuring of Superficies and Solids, Gauging, *&c.* Likewiſe Mr. *Hunt* has written largely of the Uſes of his Sliding-Rule, in Arithmetick, Geometry, Trigonometry, Gauging, Dialling, *&c.* There are ſeveral others who have explained the Uſe of their own Rules; ſo that the more curious Readers may find full Satisfaction in thoſe Authors.

One thing I have omitted in the Book, which I think may not be very improperly inſerted in this Place; that is, how to find a Number upon the Line. If the Number you would find conſiſts only of Units, then the Figures upon the Line repreſent the Number ſought: Thus, if the Number be 1, 2, 3, *&c.* then 1, 2, 3, *&c.* upon the Line, repreſents the Number ſought. But if the Number conſiſts of two Figures, that is, of Units and Tens, then the Figure upon the Rule ſtands for

the Tens, and the large Divisions stand for the Units; thus, if 34 were to be found upon the Line, the Figure 3 upon the Line is 30, and 4 of the large Divisions (counted forwards) is the Point representing 34; and if 340 were to be found, it will be at the same Point upon the Line; and if 304 were to be found, then the 3 upon the Line is 300, and four of the smaller Divisions (counted forward) is the Point representing 304. If the Number consists of four Places, or Thousands, then the Figure upon the Line stands for Thousands and the larger Divisions are Hundreds, the lesser Divisions are Tens, and the tenth Parts of those lesser Divisions are Units. Thus, if 2735 were to be found, then the two is 2000; and the 7 larger Divisions (counted forward) is 700 more; and 3 of the lesser Divisions is 30 more; and half of one of the lesser Divisions is 5 more, which is the Point representing 2735. You must remember, that between each Figure upon the Line there are 10 Parts, which I call the larger Divisions; and each of those larger Divisions are subdivided (or supposed so to be) into 10 other Parts, which I call the smaller Divisions; and each of those Parts supposed to be subdivided again into ten other Parts, *&c.* You must also remember, that if 1 in the Middle of the Line stands only for 1, then 1 at the upper End will be 10, and 1 at the lower End will only be $\frac{1}{10}$; but if 1 at the lower End signifies 1, then 1 in the Middle stands for 10, and 1 at the upper End is 100, *&c.*

There is one Thing more which I would have my Reader to understand; and that is, how to find

all

all ſuch proportional Numbers made uſe of in the Proportions about a Circle, and of a Cylinder, and in other Places; which Thing may be of good Uſe to know how to correct a Number, which may happen to be falſe printed, or to inlarge any Number to more decimal Places, for more Exactneſs; for tho' I have mentioned what ſuch Numbers are, yet I have not ſhewn how to find them, which a Learner may be a little at a Nonplus to do; tho' they are eaſily found by the Rules there laid down. I ſhall therefore give two or three Examples, in this Place, of finding ſuch Numbers, which may enable my Reader to find out the reſt.

And, firſt, let it be required to find the Area of a Circle, whoſe Diameter is an Unit.

By the Proportion of *Van Culen*, if the Diameter be 1, the Circumference will be 3.1416926, *&c.* whereof 3.1416 is ſufficient in moſt Caſes. Then the Rule teaches to multiply half the Circumference by half the Diameter, and the Product is the Area: That is, multiply 1.5708 by 5 (*viz.* half 3.1416 by half 1) and the Product is .7854, which is the Area of the Circle, whoſe Diameter is 1.

Again; if the Area be required when the Circumference is 1, firſt find what the Diameter will be, thus: 3.1416 : to 1 : : ſo is 1 to .318309, which is the Diameter when the Circumference is 1. Then multiply half .318309 by half 1; that is .159154 by .5, and the Product is .079577, which is the Area of a Circle whoſe Circumference is 1.

If

If the Area be given, to find the Side of the Square equal, you need but extract the Square Root of the Area given, and it is done: So the Square Root of .7854 is .88.62, which is the Side of a Square equal when the Diameter is 1. And if you extract the Square Root of .079577, it will be .2821, which is the Side of the Square equal to the Circle whose Circumference is 1.

If the Side of a Square within a Circle be required, if you square the Semidiameter, and double that Square, and out of that Sum extract the Square Root, that shall be the Side of the Square which may be inscribed in that Circle; so if the Diameter of the Circle be 1, then the half is .5; which squared, is .25; and this, doubled, is 5, whose Square Root is .7071, the Side of the Square inscribed.

Again; If the Diameter of a Globe be 1, to find the Solidity. In Sect. XI. Chap. II. it is demonstrated, that the Globe is $\frac{2}{3}$ of a Cylinder of the same Diameter and Altitude: Thus, if the Cylinder's Diameter be 1, and its Altitude or Length be also 1, find the Solidity thereof, and take $\frac{2}{3}$ of it, and that will be the Solidity of the Globe required. Now if the Diameter be 1, the Area of the Circle, or Base of the Cylinder, is .7854 (as is above shewn) which multiplied by 1, the Altitude of the Cylinder, and the Product is also .7854, the Solidity of the Cylinder; $\frac{2}{3}$ whereof is .5236, which is the Solidity of the Globe, whose Diameter is 1.

From

From what has been ſaid, the Reader may eaſily perceive how all other proportional Numbers are found, and may examine them at his Pleaſure.

I ſhall not inlarge any farther upon the Matter, but leave the Book to ſpeak for itſelf; and if it prove beneficial to the ingenious Practitioners, I have my Deſire. So, wiſhing my ingenious Reader good Succeſs in his Endeavours, not doubting but he will reap Profit hereby; which that he may, is the hearty Deſire of his Well-wiſher,

W. HAWNEY.

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THE

THE
Complete MEASURER.

PART I.

CHAP. I.
Notation of DECIMALS.

A DECIMAL Fraction is an artificial Way of ſetting down and expreſſing of Natural or Vulgar Fractions, as whole Numbers: And whereas the Denominators of Vulgar Fractions are divers, the Denominators of Decimal Fractions are always certain: For a Decimal Fraction hath always for its Denominator an Unit, with a Cypher or Cyphers annexed to it, and muſt therefore be either 10, 100, 1000, 10000, *&c.* and therefore, in writing down of a Decimal Fraction, there is no Neceſſity of writing down the Denominator; for by bare Inſpection it is certainly known, it conſiſting of an Unit with as many Cyphers annexed to it as there are Places (or Figures) in the Numerator.

 Example.

Example. This Decimal Fraction $\frac{25}{100}$ may be written thus .25, its Denominator being known to be an Unit with two Cyphers; becauſe there are two Figures in the Numerator. In like manner, $\frac{125}{1000}$ may be thus written, .125; and $\frac{3575}{10000}$ thus, .3575; and $\frac{75}{1000}$ thus, .075; and $\frac{65}{10000}$ thus, .0065.

As whole Numbers increaſe in a decuple or ten-fold Proportion, towards the Left Hand, ſo, on the contrary, Decimals decreaſe towards the Right Hand in a decuple Proportion, as in the following Scheme.

Tens of Millions.	Millions.	Hundreds of Thouſands.	Tens of Thouſands.	Thouſands.	Hundreds.	Tens.	Units.	Tenth Parts.	Hundredth Parts.	Thouſandth Parts.	Ten Thouſandth Parts.	Hundred Thouſandth Parts.	Millionth Parts, &c.
7	6	5	4	3	2	1	0	1	2	3	4	5	6

Hence it appears, that Cyphers put on the Right Hand of whole Numbers, do increaſe the Value of thoſe Numbers in a decuple (or ten-fold) Proportion; but being annexed to the Right Hand of a Decimal Fraction, do neither increaſe nor decreaſe the Value thereof: So $\frac{2500}{10000}$ is equivalent to $\frac{025}{100}$ or .25. And, on the contrary, tho' in whole Numbers, Cyphers prefixed before them, do neither increaſe nor diminiſh the Value; yet Cyphers before a Decimal Fraction do diminiſh its Value in a decuple Proportion: For .25, if you prefix a Cypher before it, becomes $\frac{025}{1000}$ or .025: And .125 is $\frac{00125}{100000}$, by prefixing two Cyphers before it, thus, .00125. And therefore, when you are to write a Decimal Fraction, whoſe Denominator hath more Cyphers than there are Figures in the Numerator,

merator, they must be supplied by prefixing so many Cyphers before the Figures of your Numerator; as, suppose $\frac{19}{1000}$ were to be written down, without its Denominator; here, because there are three Cyphers in the Denominator, and but two Figures in the Numerator, therefore prefix a Cypher before 19, and set it down thus, .019.

The Integers are separated from the Decimals several Ways, according to Mens Fancies; but the best and most usual Way is by a Point or Period; and if there be no whole Number, then a Point before the Fraction is sufficient: Thus, if you were to write down 317 $\frac{217}{1000}$, it may be thus expressed, 317.217; and 59 $\frac{25}{10000}$ thus, 59.0025; and $\frac{75}{10000}$ thus, .0075, *&c.*

CHAP. II.

Reduction of DECIMALS.

IN *Reduction of Decimals*, there are three Cases: 1st, To reduce a Vulgar Fraction to a Decimal. 2dly, To find the Value of a Decimal in the known Parts of Coin, Weights, Measures, *&c.* 3dly, To reduce Coin, Weights, Measures, *&c.* to a Decimal. Of these in their Order.

I. *To reduce a Vulgar Fraction to a Decimal.*

The RULE.

As the Denominator of the given Fraction is to its Numerator, so is an Unit (with a competent Number of Cyphers annexed) to the Decimal required.

Therefore, if to the Numerator given, you annex a competent Number of Cyphers, and divide the Re-

ſult by the Denominator, the Quotient is the Decimal equivalent to the Vulgar Fraction given.

Example 1. Let $\frac{3}{4}$ be given, to be reduced to a Decimal of two Places, or having 100 for its Denominator.

To 3 (the Numerator given) annex two Cyphers, and it makes 300; which divide by the Denominator 4, and the Quotient is .75, the Decimal required, and is equivalent to $\frac{3}{4}$ given.

Note, That ſo many Cyphers as you annex to the given Numerator, ſo many Places muſt be pricked off in the Decimal found; and if it ſhall happen, that there are not ſo many Places of Figures in the Quotient, the Deficiency muſt be ſupplied, by prefixing ſo many Cyphers before the Quotient Figures, as in the next Example.

Example 2. Let $\frac{3}{573}$ be reduced to a Decimal having ſix Places.

To the Numerator annex ſix Cyphers, and divide by the Denominator, and the Quotient is 5235; but it was required to have ſix Places, therefore you muſt prefix two Cyphers before it, and then it will be 005235, which is the Decimal required, and is equivalent to $\frac{3}{573}$.

See the Work of theſe two Examples.

```
4)3.00(.75          573)3.000000(005235
  28                      ....
  ——                     ——————
   20                    1350
   20                     2040
  ——                       3210
   ..                     —————
                            345
```

In the ſecond Example there remains 345, which Remainder is very inſignificant, it being leſs than $\frac{1}{100000}$ Part of an Unit, and therefore is rejected.

II. *To*

II. *To find the Value of a Decimal in the known Parts of Money, Weight, Measures,* &c.

The RULE.

Multiply the given Decimal by the Number of Parts in the next inferior Denomination, and from the Product prick off so many Places to the Right Hand as there were Places in the Decimal given; and multiply those Figures pricked off by the Number of Parts in the next inferior Denomination, and prick off so many Places as before, and so continue to do, till you have brought it to the lowest Denomination required.

Example 1. Let .7565 of a Pound Sterling be given to be reduced to Shillings, Pence, and Farthings.

Multiply by 20, by 12, and by 4, as the Rule directs, and always prick off four Places to the Right Hand, and you will find it make 15*s.* 1*d.* 2*q.* See the Work.

$$\begin{array}{rl} & .7565 \\ & 20 \\ \hline s. & \\ & 15.1300 \\ & 12 \\ \hline d. & \\ & 1.5600 \\ & 4 \\ \hline q. & \\ & 2.2400 \end{array}$$

A more compendious Way of finding the Value of the Decimal of a Pound Sterling.

Double the first Figure, (or Place of Primes) and it makes so many Shillings; and if the next Figure (or Place of Seconds) be 5, or more than 5, for the 5 add another Shilling to the former Shillings; then for

for every Unit in the ſecond Place count ten, and to that add the Figure in the third Place, and reckon that ſo many Farthings; but if they make above 13, abate 1; and if it be above 38, abate 2, and add the remaining Farthings to the Shillings before found.

Example 1. Let .695 of a Pound be reduced to Shillings, Pence, and Farthings.

Firſt, Double your 6, and it makes 12*s.* then take 5 out of 9, and for that reckon another Shilling, and it makes 13*s.* and the 4 remaining is four Tens, and the 5 makes 45, which being above 38, you muſt therefore caſt away 2, and there reſt 43 Farthings, which is 10*d.* $\frac{3}{4}$. So the Anſwer is 13*s.* 10*d.* $\frac{3}{4}$.

	l.	*s.*	*d.*
So the Value of .725=		14	6
And the Value of .878=		17	6$\frac{3}{4}$
And the Value of .417=		8	4

And ſo of any other.

Let .59755 of a Pound Troy be reduced to Ounces, Penny-weights, and Grains.

Multiply by 12, by 20, and by 24, and always prick off five Places towards the Right Hand, and you will find the Anſwer to be 7 *oz.* 3 *pwt.* 13 *gr.* *ferè.* See the Work.

$$\begin{array}{r} .59755 \\ 12 \\ \hline 7.17060 \\ 20 \\ \hline 3.41200 \\ 24 \\ \hline 164800 \\ 82400 \\ \hline 9.88800 \end{array}$$

	oz.	*pwt.*	*gr.*
Facit	7	3	9.888

Let

Let .43569 of a Ton be reduced to Hundreds, Quarters, and Pounds.

Multiply by 20, by 4, and by 28, and the Answer will be 8 *C.* 2 *qrs.* 24 *lb. ferè.*

```
 .43569
     20
 -------
 8.71380
       4        C. qrs.  lb.
 ------- Facit  8   2   23.9456
 2.85520
      28
 -------
23.94560
```

Let .9595 of a Foot be reduced into Inches and Quarters.

```
 .9595
    12
 -------
11.5140
     4  Facit 11 Inches, 2 Quarters.
 -------
 2.0560
```

III. *To reduce the known Parts of Money, Weight, Measure, &c. to a Decimal.*

The RULE.

To the Number of Parts of the lesser Denomination given, annex a competent Number of Cyphers, and divide by the Number of such Parts that are contained in the greater Denomination, to which the Decimal is to be brought; and the Quotient is the Decimal sought.

Example

Example 1. Let 6 *d.* be reduced to the Decimal of a Pound.

To 6 annex a competent Number of Cyphers (suppose 3), and divide the Result by 240 (the Pence in a Pound), and the Quotient is the Decimal required.

240)6.00|0(.025

1200

Facit .025

....

Example 2. Let 3 *d.* $\frac{3}{4}$ be reduced to the Decimal of a Pound, having six Places.

In 3 *d.* $\frac{3}{4}$ there are fifteen Farthings, therefore to 15 annex six Cyphers (because there are to be six Places in the Decimal required), and divide by 960 (the Farthings in a Pound), and the Quotient is .015625.

96|0)15.00000|0(.015625

.....

540
600
240
480

...

Example 3. Let 3 $\frac{1}{4}$ Inches be reduced to the Decimal of a Foot, consisting of four Places.

In 3 $\frac{1}{4}$ Inches, there are 13 Quarters; therefore to 13 annex four Cyphers, and divide by 48 (the Quarters in a Foot), and the Quotient is .2708.

48)13.0000(.2708

340
400

16

Example

Example 4. Let 9 *C.* 1 *qr.* 16 *lb.* be reduced to the Decimal of a Ton, having six Places.

C. qu. lb.	
9 1 16	2240)1052.00000|0(469642
4	......
37 *qrs.*	15600
28	21600
	14400
302 *Facit* .469642	9600
75	6400
1052 Pounds.	1920

CHAP. III.

Addition of DECIMALS.

ADDITION of Decimals is performed the same Way as Addition of whole Numbers, only you must observe to place your Numbers right, that is, Units under Units, Primes under Primes, Seconds under Seconds, &c.

Example. Let 317.25, 17.125, 275.5, 47.3579, and 12.75, be added together into one Sum.

```
 317.25
  17.125
 275.5
  47.3579
  12.75
 --------
Sum 669.9829
```

This is so plain, that more Examples I think needless.

CHAP.

CHAP. IV.

Subtraction of DECIMALS.

SUBTRACTION of Decimals is performed likewise the same Way as in whole Numbers, respect being had to the right placing the Numbers (as in Addition), as in the following Examples.

	(1)		(2)
From	212.0137	From	201.1250
Subtr.	31.1275	Subtr.	5.5785
Rests	180.8862	Rests	195.5465
Proof	212.0137	Proof	201.1250

	(3)		(4)
From	2051.315	From	30.5
Subtr.	79.172	Subtr.	7.2597
Rests	197.2143	Rests	23.2403
Proof	2051.315	Proof	30.5

Note, If the Number of Places in the Decimals be more in that which is to be subtracted, than in that which you subtract from, you must suppose Cyphers to make up the Number of Places, as in the fourth Example.

CHAP.

CHAP. V.

Multiplication of DECIMALS.

MULTIPLICATION of Decimals is alſo performed the ſame Way as Multiplication of whole Numbers; but to know the Value of the Product, obſerve this Rule.

Cut off, or ſeparate by a Comma or Prick, ſo many decimal Places in the Product, as there are Places of Decimals in both Factors, *viz.* in the Multiplicand and Multiplier; which I ſhall farther explain in the following Examples.

Let 3.125 be multiplied by 2.75; multiply the Numbers together, as if they were whole Numbers, and the Product is 8.59375: and becauſe there were three Places of Decimals pricked off in the Multiplicand, and two Places in the Multiplier, therefore you muſt prick off five Places of Decimals in the Product, as you may ſee by the Work.

$$
\begin{array}{r}
3.125 \\
2.75 \\
\hline
15625 \\
21875 \\
6250 \\
\hline
8.59375
\end{array}
$$

Let 79.25 be multiplied by .459.

In this Example, becauſe two Places of Decimals are pricked off in the Multiplicand, and three in the Multiplier, therefore there muſt be five pricked off in the Product.

```
     79.25
      .459
  --------
     71325
    39625
   31700
  --------
  36.37575
```

Let .135272 be multiplied by .00425.

In this Example, becauſe in the Multiplicand are ſix decimal Places, and in the Multiplier five Places; therefore in the Product there muſt be eleven Places of Decimals; but when the Multiplication is finiſhed, the Product is but 57490600, *viz.* only eight Places; therefore, in this Caſe, you muſt prefix three Cyphers before the Product Figures, to make up the Number of eleven Places: ſo the true Product will be .00057490600.

```
       .135272
        .00425
  ------------
        676360
       270544
      541088
  ------------
  .00057490600
```

More

More Examples for Practice.

.001472	.017532
.1045	347
7360	122724
5888	70128
14720	52596
.0001538240	6.083604

279.25	32.07552
.445	.0325
139625	1603760
111700	641504
111700	962256
124.26625	1.04244400

4.443	20.0291
15.98	35.45
35544	1001455
39987	801164
22215	1001455
4443	600873
70.99914	710.031595

7.3564	.75432
.0126	.0356
441384	452592
147128	377160
73564	226296
.09269064	.026853792

Contracted Multiplication of Decimals.

Becauſe in Multiplication of Decimal Parts, and mixed Numbers, there is no Need to expreſs all the Figures of the Product, but in moſt Places two, three, or four Places of Decimals will be ſufficient; therefore, to contract the Work, obſerve the following

RULE.

Write the Unit's Place of the Multiplier under that Place of the Multiplicand, whoſe Place you intend to keep in the Product; then invert the Order of all the other Figures; that is, write them all the contrary Way. Then, in multiplying, always begin at that Figure in the Multiplicand which ſtands over the Figure you are then multiplying withal, and ſet down the firſt Figure of each particular Product directly one under the other: But yet a due Regard muſt be had to the Increaſe ariſing from the Figures on the Right Hand of that Figure in the Multiplicand which you begin to multiply at. This will appear more plain by Examples.

Example 1. Let 2.38645 be multiplied by 8.2175, and let there be only four Places retained in the Decimals of the Product.

Firſt, according to the Directions, write down the Multiplicand, and under it write the Multiplier, thus; place the 8 (being the Unit's Place of the Multiplier) under 4, the fourth Place of Decimals in the Multiplicand, and write the reſt of the Figures quite contrary to the uſual Way, as in the following Work: Then begin to multiply, firſt the 5 which is left out (only with regard to the Increaſe which muſt be carried from it); ſaying, 8 times 5 is 40; carry 4 in your Mind, and ſay, 8 times 4 is 32, and 4 I carry, is 36; ſet down 6, and carry 3, and proceed thro' the reſt of the Figures as in common Multiplication: Then

Then begin to multiply with 2; ſaying, 2 times 4 is 8, for which I carry 1 (becauſe it is above 5), and ſay, 2 times 6 is 12, and 1 that I carry is 13; ſet down 3, and carry 1, and proceed thro' the reſt of the Figures: Then multiply with 1; ſaying, once 6 is 6, for which carry 1, and ſay, once 8 is 8, and 1 is 9; ſet down 9, and proceed: Then multiply with 7; ſaying, 7 times 8 is 56, for which carry 6 (becauſe it is above 55), and ſay, 7 times 3 is 21, and 6 that I carry is 27; ſet down 7, and carry 2, and proceed: Then multiply with 5; ſaying, 5 times 3 is 15, for which carry 2, and ſay, 5 times 2 is 10, and 2 I carry is 12, which ſet down, and add all the Products together; and the total Product will be 19.6107. See the Work.

```
 2.38645
 5712.8
 -------
 19.0916
    4773
     239
     167
      12
 -------
 19.6107
```

Note, That in multiplying the Figure left out every Time next the Right Hand in the Multiplicand, if the Product 5, or upwards to 10, you carry 1; and if it be 15, or upwards to 20, carry 2; and if 25, or upwards to 30, carry 3, &c.

I have here ſet down the Work of the laſt Example, wrought by the common Way, by which you may ſee both the Reaſon and Excellency of this Way, all the Figures on the Right Hand of the Line being wholly omitted.

$$
\begin{array}{r|l}
2. & 38645 \\
8. & 2175 \\
\hline
11 & 93225 \\
167 & 0515 \\
238 & 645 \\
4772 & 90 \\
190916 & 0 \\
\hline
19.6106 & 52875
\end{array}
$$

Example 2. Let 375.13758 be multiplied by 16.7324, ſo that the Product may have but four Places of Decimals.

Firſt, ſet 6, the Unit's Place of the Multiplier, under 5, being the fourth Place of Decimals in the Multiplicand (becauſe four Places of Decimals were to be pricked off), and write all the reſt of the Figures backward. Then multiply all the Figures of the Multiplicand by 1, after the common Way. Then begin with the ſecond Figure of the Multiplier 6; ſaying, 6 times 8 is 48, for which I carry 5 (in reſpect of the 8 left out), and 6 times 5 is 30, and 5 that I carry is 35; ſet down 5 and carry 3, and proceed after the common Method. Then begin with 7, the third Figure of the Multiplier, and ſay 7 times 5 is 35, for which carry 4, and ſay 7 times 7 is 49, and 4 I carry is 53; ſet down 3 under the firſt, and carry 5, and proceed as before. Then begin with 3, the fourth Figure of the Multiplier, and ſay 3 times 7 is 21, carry 2, and ſay 3 times 3 is 9, and 2 I carry is 11; ſet down 1 and carry 1, and proceed as before. Then begin with 2, the fifth Figure, and ſay 2 times 3 is 6, for which I carry 1, and ſay 2 times 1 is 2, and 1 I carry is 3; ſet down 3, and 2 times 5 is 10; ſet down 0, and carry 1, and proceed as before. Then begin with 4, the laſt Figure of the Multiplier, and ſay 4 times 1 is 4, for which I carry nothing, becauſe it is leſs than 5: Then ſay 4 times

times 5 is 20; ſet down 0, and carry 2, and proceed thro' the reſt of the Figures of the Multiplicand. Then add all up together, and the Product is 6276.9520. See the Work.

375.13758 the Multiplicand.
4237.61 the Multiplier reverſed.

37513758 the Product with 1.
22508255 the Product with 6 increaſed with 6 × 8.
2625963 the Product with 7 increaſed with 7 × 5.
112541 the Product with 3 increaſed with 3 × 7.
7503 the Product with 2 increaſed with 2 × 3.
1500 the Product with 4 increaſed with 0.

6276.9520 the Product required.

Let the ſame Example be repeated, and let only one Place in Decimals be pricked off.

375.13758 the Multiplicand.
4237.61 the Multiplier reverſed.

37514 the Product by 1 with the Increaſe of 1 × 7.
22508 the Product with 6 increaſed with 6 × 3.
2626 the Product with 7 increaſed with 7 × 1.
113 the Product with 3 increaſed with 3 × 5.
7 the Product with 2 increaſed with 2 × 7.
1 the Increaſe only of 4 × 3.

6276.9 the Product is the ſame as before.

More Examples for Practice.

Multiply 395·3756 by .75642; and prick off four Places in Decimals.

395·3756 the Multiplicand.
24657. the Multiplier reversed.

2767629 the Product by 7 increased with 7 × 6.
197688 the Product by 5 increased with 5 × 5.
23722 the Product by 6 increased with 6 × 7.
1581 the Product by 4 increased with 4 × 3.
79 the Product by 2 increased with 2 × 5.

299.0699 the Product required.

Let the same Example be repeated, and let there be only one Place of Decimals.

395·3756
24657.

2767 the Product by 7 increased with 7 × 3.
198 the Product by 5 increased with 5 × 5.
24 the Product by 6 increased with 6 × 9 + 6 × 5.
2 the Increase of 4 × 9 + 4 × 3.

299.1 the Product.

Characters,

Characters, and their Signification.

Note, That this Mark + ſignifies Addition; as 8+5, that is, 8 more 5, or 8 added to 5; and 8+3 +7, denotes theſe Numbers are to be added into one Sum.

This Mark — ſignifies Subtraction, as 9—4 ſignifies that 4 is to be taken from 9.

This Mark × ſignifies Multiplication, as 7 × 5 ſignifies that 7 is to be multiplied by 5.

This Mark ÷ ſignifies Diviſion, as 12 ÷ 4 ſignifies 12 is to be divided by 4.

This Mark = ſignifies Equality, or Equation; that is, when = is placed between Numbers, or Quantities, it denotes them to be equal, as 7+5 = 12, that is, 7 more 5 is equal to 12; and 15 — 7 = 8, that is, 15 leſs by 7, is equal to 8, or ſubtract 7 from 15 and there remains 8.

This Mark : : is the Sign of Proportion, or the Golden Rule, it being always placed betwixt the two middle Terms or Numbers in Proportion; thus, 4 : 20 : : 6 : 30, to be thus read, as 4 is to 20, ſo is 6 to 30.

CHAP. VI.

Diviſion of DECIMALS.

DIVISION of Decimals is performed after the ſame Manner as Diviſion of whole Numbers; but to know the Value or Denomination of the Quotient, is the only Difficulty; for the reſolving of which, obſerve either of the following

RULES.

RULES.

I. The first Figure in the Quotient must be of the same Denomination with that Figure in the Dividend which stands (or is to be supposed to stand) over the Unit's Place in the Divisor, at the first Seeking.

II. When the Work of Division is ended, count how many Places of Decimal Parts there are in the Dividend more than in the Divisor; for that Excess is the Number of Places which must be separated in the Question for Decimals: But if there be not so many Figures in the Quotient as is the said Excess, that Deficiency must be supplied with Cyphers in the Quotient, prefixed before the significant Figures thereof, towards the Left Hand, with a Point before them; so shall you plainly discover the Value of the Quotient.

These following Directions ought also to be carefully observed.

If the Divisor consists of more Places than the Dividend, there must be a competent Number of Cyphers annexed to the Dividend, to make it consist of as many (at least) or more Places of Decimals than the Divisor; for the Cyphers added must be reckoned as Decimals.

Consider whether there be as many Decimal Parts in the Dividend as there are in the Divisor; if there be not, make them so many, or more, by annexing of Cyphers.

In dividing of whole or mixed Numbers, if there be a Remainder, you may bring down more Cyphers; and, by continuing your Division, carry the Quotient to as many Places of Decimals as you please.

These Things being considered, I shall proceed to the Practice of Division of Decimals, which I shall endeavour to explain in as familiar and easy a Method as possible.

Example

Example 1. Let 48 be divided by 144.

In this Example the Divisor 144 is greater than the Dividend 48; therefore, according to the Directions above, I annex a competent Number of Cyphers (*viz.* four), with a Point between them, and divide according to the usual Way.

```
144)48.0000(.3333
    -------
    480
     480
      480
    -------
       48
```

But, first, in seeking how often 144 in 48.0 (the first three Figures of the Dividend), I find the Unit's Place of the Divisor to fall under the first Place of Decimals; therefore the first Figure in the Quotient is in the first Place of Decimals: Or, by the second Rule, there being four Places of Decimals in the Dividend, and none in the Divisor; so the Excess of decimal Places in the Dividend, above that in the Divisor, is four; so that when the Division is ended, there must be four Places of Decimals in the Quotient. See the Work.

Example 2. Let 217.75 be divided by 65.

First, in seeking how often 65 in 217 (the first three Figures of the Dividend), I find the Unit's Place of the Divisor to fall under the Unit's Place of the Dividend; therefore the first Figure in the Quotient will be Units, and all the rest Decimals: Or, by the second Rule, there being two Places of Decimals in the Dividend, and no Decimals in the Divisor, therefore the Excess of Decimal Places in the Dividend, above the Divisor, is two; so when the Division is ended, separate two Places in the Quotient, towards the Right Hand by a Point. See the Work.

65)

65)217.75(3.35
...

227
325

...

Example 3. Let 267.15975 be divided by 13.25.

13.25)267.15975(20.163
.....

2159
8347
3975

....

In this Example, 3, the Unit's Place of the Divisor, falls under 6, the Ten's Place of the Dividend; therefore (by the first Rule) the first Figure in the Quotient is Tens: Or, by the second Rule, the Excess of Decimal Places in the Dividend, above the Divisor, is three; there being five Places of Decimals in the Dividend, and but two in the Divisor, so there must be three Places of Decimals in the Quotient.

Example 4. Let 15.675159 be divided by 375.89.

375.89)15.675159(.0417
..

63955
263669

546

In

In this Example, 5, in the Unit's Place of the Diviſor, falls under 7, the ſecond Place of Decimals in the Dividend; therefore (by the firſt Rule) the firſt Figure in the Quotient is in the ſecond Place of Decimals; ſo that you muſt put a Cypher before the firſt Figure in the Quotient; and by the ſecond Rule, the Exceſs of decimal Places in the Dividend above the Number of decimal Places in the Diviſor is 4; for the decimal Places in the Dividend is 6, and the Number of Places in the Diviſor but two; therefore there muſt be four Places of Decimals in the Quotient: But the Diviſion being finiſhed after the common Way, the Figures in the Quotient are but three; therefore you muſt prefix a Cypher before the ſignificant Figures.

Example 5. Let 72.1564 be divided by .1347.

```
.1347)72.1564(535.68
        . . .
     ---------
        4806
         7654
          9190
           11080
          -------
             304
```

In this Example, the Diviſor being a Decimal, the firſt Figure thereof falls under the Ten's Place in the Dividend; therefore the Units (if there had been any) ſhould fall under the Hundred's Place in the Dividend, and ſo the firſt Figure in the Quotient is Hundreds. And, by the ſecond Rule, there being four Places of Decimals in the Dividend, and as many in the Diviſor, ſo the Exceſs is nothing; but in dividing I put two Cyphers to the Remainders, and continue the Diviſion to two Places further; ſo I have two Places of Decimals. See the Work.

Example 6. Let .125 be divided by .0457.

```
.0457).1250000(2.735
      ....
   ---------
    914
    ---
    3360
    3199
    ----
     1610
     1371
     ----
      3390
      2285
      ----
       105
```

In this Example, the Unit's Place of the Divisor (if there had been any) would fall under the Unit's Place of the Dividend; therefore the first Figure of the Quotient is Units. And, by the second Rule, there being seven Places of Decimals in the Dividend, and but four Places in the Divisor, so the Excess is three; therefore there must be three Places of Decimals in the Quotient.

I shall set down only the Work of some few Examples more, and so proceed to *Contracted Division.*

```
.00456).0000059791(.00131
               ..
          --------
             1419
              511
             ----
               55
```

Let

Let it be divided by 282.

```
282)1.0000000(.0035461 ferè.
       .....
     ________
     1540
      1300
       1720
        280
```

```
.325).400000(1.2307
           ...
       ________
       750
       1000
        2500
       ________
         225
```

```
.042)495.00000(11785.71
          ......
       ________
       75
       330
        360
         240
          300
           60
       ________
           18
```

Division of DECIMALS *contracted.*

IN Division of Decimals the common Way, when the Divisor hath many Figures, and it is required to continue the Division till the Value of the Remainder be but small, the Operation will sometimes be large and tedious, but may be excellently contracted by the following Method.

The RULE.

By the first Rule of this Chapter (*Page* 20), find what is the Value of the first Figure in the Quotient; then, by knowing the first Figure's Denomination, you may have as many or as few Places of Decimals as you please, by taking as many of the Left Hand Figures of the Divisor as you think convenient for the first Divisor; and then take as many Figures of the Dividend as will answer them; and, in dividing, omit one Figure of the Divisor at each following Operation. A few Examples will make it plain.

Example 1. Let 721.17562 be divided by 2.25743[illegible]; and let there be three Places of Decimals in the Quotient.

```
2.25743)721.175|62(319.467
.....   677229
        ------
         43946
         22574
        ------
         21372
         20317
        ------
          1055
           903
        ------
           152
           135
        ------
            17
            15
        ------
             2
```

In this Example, the Unit's Place of the Divisor falls under the Hundred's Place in the Dividend, and it is required, that three Places of Decimals be in the Quotient, so there must be six Places in all; that is, three Places of whole Numbers, and three Places of Decimals. Then, because I can have the Divisor in the first six Figures of the Dividend, I cut off the 62 with a Dash of the Pen, as useless; then I seek how often the Divisor in the Dividend, and the Answer is three times; put 3 in the Quotient, and multiply and subtract as in common Division, and the Remainder is 43946. Then prick off the 3 in the Divisor, and seek how often the remaining Figures may be had in 43946, the Remainder, which can be but once; put 1 in the Quotient, and multiply and subtract, and the next Remainder is 21372. Then prick off the 4 in the Divisor, and seek how often the remaining Figures may be had in 21372, which will be 9 times; put 9 in the Quotient; multiply thus, saying 9

times 4 is 36, for which I carry 4 (in reſpect of the 4 laſt pricked off), and 9 times 7 is 63, and 4 is 67; ſet down 7, and carry 6, and ſo proceed till the Diviſion be finiſhed, always reſpecting the Increaſe made from the Figures pricked off. Obſerve the Work, which will better inform you than many Words.

2.25743)721.17562(319.467

677229	
43946	6
22574	3
21372	32
20316	87
1055	450
902	972
152	4780
135	4458
17	03220
15	80201
1	23019

I have ſet down the Work of this laſt Example at large, according to the common Way, that thereby the Learner may ſee the Reaſon of the Rule, all the Figures on the Right Side the perpendicular Line being wholly omitted.

Example

Example 2. Let 5171.59165 be divided by 8.758615; and let it be required, that four Places of Decimals be pricked off in the Quotient.

```
8.758615)5171.5916|5(590.4577
. . . . . .
         43793075
         --------
          7922841
          7882754
          -------
            40087
            35034
            -----
             5053
             4379
             ----
              674
              613
              ---
               61
               61
               --
               ..
```

In this Example, I can't have 8, the first Figure in the Divisor, in 5, the first Figure of the Dividend; so that the Unit's Place of the Divisor falls under the Hundred's Place of the Dividend; so that there will be seven Figures in the Quotient; that is, three of whole Numbers, and four of Decimals; therefore there must be seven Figures in the Divisor (because the Number of Places in the Divisor and Quotient will be equal), and there must be eight Places in the Dividend; so that I cut off the Figure 5 with a Dash, as useless. Thus having proportioned the Dividend to the Divisor, and both to the Number of Places or Figures desired in the Quotient, I proceed to divide as before; saying how often 8 in 51, which will be 5 times; put 5 in the Quotient, and multiply and subtract,

tract, and the Remainder is 7922841. Then I prick off the first Figure in the Divisor, 5, and seek how often the remaining Figures of the Divisor in the aforesaid Remainder, which I find nine times; put 9 in the Quotient, and multiply thereby, saying 9 times 5 (the Figure pricked off) is 45, for which I carry 5, and say 9 times 1 is 9, and 5 I carry is 14; set down 4, and carry 1, and proceed to multiply the rest of the Figures, and subtract, and the Remainder will be 40087. Then prick off the Figure 1, and seek how often 87586 in the Remainder 40087, the Answer will be 0; so put 0 in the Quotient, and prick off the Figure 6, and seek how often 8758 in 40087, which will be four times; put 4 in the Quotient, and multiply, saying 4 times 6 (the Figure last pricked off) is 24, for which I carry 2, and say 4 times 8 is 32, and 2 I carry is 34; set down 4, and carry 3; multiply the rest of the Figures, and subtract as before, and so proceed after the same manner, until all the Figures of the Divisor be pricked off, to the last Figure. See the Work.

Example 3. Let 25.1367 be divided by 217.3543, and let there be five Places of Decimals in the Quotient.

In this Example, 7, the Unit's Place of the Divisor, falls under 1, the first Place of Decimals; therefore the first Figure of the Quotient is in the first Place of Decimals; so the Quotient will be all Decimals. Then, because the Quotient Figures, and the Figures of the Divisor will be of an equal Number, dash off the 43 in the Divisor, and the 7 in the Dividend, as useless, and divide as before.

217.35|43

```
217.35|43)25.136|7(.11564
           21735
           -----
            3401
            2174
            ----
            1227
            1087
            ----
             140
             130
             ---
              10
               8
              --
               2
```

Altho' I have hitherto given Directions for proportioning the Divisor and Dividend, so as to bring into the Quotient what Number of Decimals you please, yet there is no absolute Necessity for it; but you may carry on your Division to what Degree you please, before you begin to prick off the Figures of the Divisor, in order to contract the Work, as in the following Examples, where it is not required to prick off any determinate Number of Decimals, but it may be done according to Discretion.

2.756756)7414.76717(2689.67118
······ ···

5513512

19012551
16540536

24720157
22054048

2666109
2481080

185029
165405

19624
19297

327
276

51
28

23
22

1

```
12.34254)514.75498(41.705757
     ... 4937016
         -------
         2105338
         1234254
         -------
          871084
          863978
          ------
            7106
            6171
            ----
             935
             864
             ---
              71
              62
              --
               9
               8
               -
               1
```

CHAP. VII.

Extraction of the SQUARE ROOT.

If a Square Number be given;

TO find the Root thereof, that is, to find out ſuch a Number, as being multiplied into itſelf, the Product ſhall be equal to the Number given; ſuch Operation is called, *The Extraction of the Square Root*; which to do, obſerve the following Directions.

1ſt,

1ſt, You muſt point your given Number; that is, make a Point or Prick over the Unit's Place, another upon the Hundred's, and ſo upon every ſecond Figure throughout.

2dly, Then ſeek the greateſt ſquare Number in the firſt Point towards the Left Hand, placing the ſquare Number under the firſt Point, and the Root thereof in the Quotient, and ſubtract the ſaid ſquare Number from the firſt Point, and to the Remainder bring down the next Point, and call that the Reſolvend.

3dly, Then double the Quotient, and place it, for a Diviſor, on the Left Hand of the Reſolvend; and ſeek how often the Diviſor is contained in the Reſolvend (reſerving always the Unit's Place), and put the Anſwer in the Quotient, and alſo on the Right Hand Side of the Diviſor; then multiply by the Figure laſt put in the Quotient, and ſubtract the Product from the Reſolvend (as in common Diviſion), and bring down the next Point to the Remainder (if there be any more), and proceed as before.

A TABLE *of* SQUARES *and* CUBES, *and their* ROOTS.

Root	1	2	3	4	5	6	7	8	9
Squ.	1	4	9	16	25	36	49	64	81
Cube	1	8	27	64	125	216	343	521	729

Example 1. Let 4489 be a Number given, and let the ſquare Root thereof be required.

4489.67
36

127.889 Reſolvend.
889 Product.

...

First,

First, Point the given Number, as before directed; then, by the little Table aforegoing, seek the greatest square Number in 44 (the first Point to the Left Hand), which you will find to be 36, and 6 the Root; put 36 under 44, and 6 in the Quotient, and subtract 36 from 44, and there remains 8. Then to that 8 bring down the other Point 89, placing it on the Right Hand, so it makes 889 for a Resolvend; then double the Quotient 6, and it makes 12; which place on the Left Hand for a Divisor, and seek how often 12 in 88 (reserving the Unit's Place), the Answer is 7 times; which put in the Quotient, and also on the Right Hand Side of the Divisor, and multiply 127 by 7, as in common Division, and the Product is 889, which subtracted from the Resolvend, there remains nothing; so is your Work finished; and the square Root of 4489 is 67; which Root if you multiply by itself, that is 67 by 67, the Product will be 4489, equal to the given square Number, and prove the Work to be right.

Example 2. Let 106929 be a Number given, and let the square Root thereof be required.

```
     . . .
     106929(.327
      9
     ------
  62)169 Resolvend.
     124 Product.
     ------
 647)4529 Resolvend.
     4529 Product.
     ------
     ....
```

First, Point your given Number, as before directed, putting a Point upon the Units, Hundreds, and Tens of Thousands; then seek what is the greatest square Number in 10 (the first Point), which by the little

Table you will find to be 9, and 3 the Root thereof; put 9 under 10, and 3 in the Quotient; then ſubtract 9 out of 10, and there remains 1; to which bring down 69, the next Point, and it makes 169 for the Reſolvend; then double the Quotient 3, and it makes 6, which place on the Left Hand of the Reſolvend for a Diviſor, and ſeek how often 6 in 16; the Anſwer is twice; put 2 in the Quotient, and alſo on the Right Hand of the Diviſor, making it 62. Then multiply 62 by the 2 you put in the Quotient, and the Product is 124; which ſubtract from the Reſolvend, and there remains 45; to which bring down 29, the next Point, and it makes 4529 for a new Reſolvend. Then double the Quotient 32, and it makes 64, which place on the Left Side of the Reſolvend for the Diviſor, and ſeek how often 64 in 452, which you'll find 7 times; put 7 in the Quotient, and alſo on the Right Hand of the Diviſor, making it 647, which multiplied by the 7 in the Quotient, it makes 4529, which ſubtracted from the Reſolvend, there remains nothing. So 327 is the Square Root of the given Number.

Example 3. Let 2268741 be a ſquare Number given, the Root whereof is required.

. . . .
2268741)1506.23
1
―――
25)126
125
―――
3006)18741
18036
―――
30122)70500
60244
―――
301243)1025600
.903729
―――
Remains .121871

Having

Having pointed the given Number as before directed, ſeek what is the greateſt ſquare Number in the firſt Point 2, which is 1; put 1, the Square, under 2, and 1, the Root thereof, in the Quotient; ſubtract 1 from 2, and there remains 1; to which bring down the next Point, 26, and ſet on the Right Hand, making it 126; double the 1 in the Quotient; which makes 2; ſet 2 on the Left Hand for a Diviſor, and aſk how often 2 in 12, which will be 5 times; put 5 in the Quotient, and alſo on the Right Hand of the Diviſor, making it 25; multiply (as in common Diviſion) 25 by 5, and ſubtract the Product 125 from 126, and there remains 1. Bring down the next Point, 87, and it makes 187 for a new Reſolvend; and double the 15 in the Quotient, it makes 30 for a new Diviſor. Then ſeek how often 30 in 18, which you can't have; ſo that you muſt put 0 in the Quotient, and alſo on the Right Hand of the Diviſor, and bring down the next Point, and it makes 18741 for another new Reſolvend. Then ſeek how often 300 in 1874, which will be 6 times; put 6 in the Quotient, and alſo on the Right Hand of the Diviſor; multiply and ſubtract, and the Remainder will be 705. Now, if you have a Mind to find the Value of the Remainder, you may annex Cyphers, by two at a time, to the Remainders, and ſo proſecute the Work to what Number of decimal Parts you pleaſe; thus, to 705 annex two Cyphers, and it will make 70500, and the Quotient doubled, is 3012 for a Diviſor: Then ſeek how often 3012 in 7050 (rejecting the Unit's Place), which will be twice; put 2 in the Quotient, and alſo on the Right Hand of the Diviſor, and multiply and ſubtract as before, and the Remainder will be 10256; to which annex two Cyphers, and proceed as before, and you will get a 3 in the Quotient next. So the ſquare Root of the given Number is 1506.23, which being ſquared, or multiplied by itſelf, and the laſt Remainder added, will make the given Number as follows.

```
                 1506.23
                 1506.23
               ---------
                  451869
                 301246
                903738
               7531150
              150623
            -----------
            2268728.8129
The Remainder add 12.1871
            -----------
     Proof 2268741.0000
```

Some more Examples for Practice.

```
                   :  .  .  .
Example 1.         7596796(2756.228 Root.
                   4
                   ----
                47)359
                   329
                   ----
               545)3067
                   2725
                   ----
              5506)34296
                   33036
                   -----
             51122)126000
                   110244
                   ------
            551242)1575600
                   1102484
                   -------
           5512448)47311600
                   44099584
                   --------
                    3212016
```

Example

```
*Example* 2.      751417.5745(866.4 Root.
                  64
                  ----
             166)1114
                  996
                  -----
            1726)11817
                 10356
                 ------
           17328)146157
                 138624
                 -------
          173364)753345
                 693456
                 -------
                  59889
                 -------
```

If the given Number be a mixed Number, *viz.* confisting of a whole Number and a Decimal together, make the Number of decimal Places even, that is, 2, 4, 6, 8, *&c.* that so there may a Point fall upon the Unit's Place of the whole Numbers, as in this last Example, and in that following.

Example 3. Let 656714.37512 be given, to find the ſquare Root.

```
            . . . . . . .
            656714.375120(810.379 Root.
            64
            ----
       161)167
           161
           ----
     16203)61437
           48609
           -----
    162067)1282851
           1134469
           -------
   1620749)14838220
           14586741
           --------
   Remains  251479
```

In this Example there are five Places of Decimals; therefore put a Cypher to it, to make it even, that ſo there may a Point fall upon 4, the Unit's Place.

To find the Square Root of a Fraction.

If it be a Decimal Fraction, the Work differs nothing from the Examples aforegoing, only you muſt be mindful to point your given Number right; for (as was before directed) the Number of Places muſt always be made even, and then begin to point at the Right Hand, as in whole Numbers.

If it be a Vulgar Fraction, it muſt be reduced to a Decimal, by the firſt Rule of the ſecond Chapter.

I ſhall give an Example or two in each Caſe, and ſo conclude this Chapter.

Let

Let .125 be a Decimal Fraction given, whose square Root is required; and let it be required to have four Places of Decimals in the Root.

```
      .  .  .  .
     .12500000(.3535
       9
      ---
   65)350
      325
     ----
  703)2500
       2109
      -----
 7065)39100
       35325
      ------
        3775
```

In this Example there must be five Cyphers annexed, because two Places in the Square make but one in the Root.

Let the square Root of .00715 be required.

```
       .  .  .
     .007150).084
        64
       ----
    164)750
        656
        ---
         94
```

In this a Cypher is added to make the Place even.

Let $\frac{7}{8}$ be a Vulgar Fraction given, whose square Root is required.

```
8)7000              (.87500000(.9354
  64                  81
  --                  --
  60              183)650
  56                  549
  --                  ---
   40            1865)10100
   40                  9325
   --                  ----
   ..           18704)77500
                      74816
                      -----
                       2684
```

Reduce this $\frac{7}{8}$ to a Decimal, it makes .875; to which annex Cyphers, and extract the square Root, as if it was a whole Number. So the Root is .9354.

Let $\frac{3}{960}$ be a Vulgar Fraction, whose square Root is required.

```
96|0)3.000000          (.00312500(.0559 Root.
     288                   25
     ---                   --
     120               105)625
      96                   525
     ---                   ---
     240              1109)10000
     192                    9981
     ---                    ----
      480                     19
      480
      ---
      ...
```

In extracting the Root of this, becauſe the firſt Point conſiſts of Cyphers, there muſt be a Cypher put firſt in the Quotient.

To prove this Rule, ſquare the Root, and to the Product add the Remainder, as was before directed. To ſquare a Number, is to multiply it by itſelf; and to cube it, is to multiply the Square of the Number by the Number itſelf.

CHAP. VIII.

Extraction of the CUBE ROOT.

TO extract the Cube Root, is nothing elſe but to find ſuch a Number, as being firſt multiplied into itſelf, and then into that Product, produceth the given Number; which to perform, obſerve the following Directions.

1*ſt*, You muſt point your given Number, beginning with the Unit's Place, and make a Point, or Dot, over every third Figure towards the Left Hand.

2*dly*, Seek the greateſt Cube Number in the firſt Point, towards the Left Hand, putting the Root thereof in the Quotient, and the ſaid Cube Number under the firſt Point, and ſubtract it therefrom, and to the Remainder bring down the next Point, and call that the Reſolvend.

3*dly*, Triple the Quotient, and place it under the Reſolvend; the Unit's Place of this under the Ten's Place of the Reſolvend; and call this the Triple Quotient.

4*thly*, Square the Quotient, and triple the Square, and place it under the triple Quotient; the Units of this

this under the Ten's Place of the triple Quotient, and call this the Triple Square.

5*thly*, Add theſe two together, in the ſame Order as they ſtand, and the Sum ſhall be the Diviſor.

6*thly*, Seek how often the Diviſor is contained in the Reſolvend, rejecting the Unit's Place of the Reſolvend (as in the Square Root), and put the Anſwer in the Quotient.

7*thly*, Cube the Figure laſt put in the Quotient, and put the Unit's Place thereof under the Unit's Place of the Reſolvend.

8*thly*, Multiply the Square of the Figure laſt put in the Quotient, into the triple Quotient, and place the Product under the laſt, one Place more to the Left Hand.

9*thly*, Multiply the triple Square by the Figure laſt put in the Quotient, and place it under the laſt, one Place more to the Left Hand.

10*thly*, Add the three laſt Numbers together, in the ſame Order as they ſtand, and call that the Subtrahend.

Laſtly, Subtract the Subtrahend from the Reſolvend, and if there be another Point, bring it down in the Remainder, and call that a new Reſolvend, and proceed in all reſpects as before.

Example

Example 1. Let 314432 be a Cubick Number, whose Root is required.

```
.  .
314432(68 Root.
216
———————
  98432 Resolvend.
———————
   18 Triple Quotient of 6.
 108  Triple Square of the Quotient 6.
————
 1098 Divisor.
————
   512 Cube of 8, the last Figure of the Root.
 1152  The Square of 8, by the triple Quotient.
 864   The triple Square of the Quotient 6 by 8.
———————
 98432 The Subtrahend.
———————
 .....
```

After you have pointed the given Number, seek what is the greatest Cube Number in 314, the first Point, which, by the former little Table (*Page* 34), you will find to be 216, which is the nearest that is less than 314, and its Root is 6; which put in the Quotient, and 216 under 314, and subtract it therefrom, and there remains 98; to which bring down the next Point, 432, and annex to 98; so will it make 98432 for the Resolvend. Then triple the Quotient 6, it makes 18, which write down the Unit's Place, 8, under 3, the Ten's Place of the Resolvend. Then square the Quotient 6, and triple the Square, and it makes 108, which write under the triple Quotient, one Place toward the Left Hand; then add those two Numbers together, and they make 1098 for the Divisor. Then seek how often the Divisor is contained in the Resolvend, (rejecting the Unit's Place thereof), that is, how often 1098 in 9843, which is

8

8 times; put 8 in the Quotient, and the Cube thereof below the Divisor, the Unit's Place under the Unit's Place of the Resolvend. Then square the 8 last put in the Quotient, and multiply 64, the Square thereof, by the triple Quotient, 18; the Product is 1152; set this under the Cube of 8, the Units of this under the Tens of that. Then multiply the triple Square of the Quotient by 8, the Figure last put in the Quotient, the Product is 864; set this down under the last Product, a Place more to the Left Hand. Then draw a Line under those three, and add them together, and the Sum is 98432, which is called the Subtrahend; which being subtracted from the Resolvend, the Remainder is nothing; which shews the Number to be a true cubick Number, whose Root is 68; that is, if 68 be cubed, it will make 314432.

For if 68 be multiplied by 68, the Product will be 4624; and this Product, multiplied again by 68, the last Product is 314432, which shews the Work to be right.

	68
	68
	544
The Work	408
	4624
	68
	36992
	27744
The Proof	314432

Example 2. Let the Cube Root of 5735339 be required.

After you have pointed the given Number, seek what is the greatest Cube Number in 5, the first Point, which (by the little Table, *Page* 34) you will find to be 1; which place under 5, and 1, the Root thereof, in

in the Quotient; and subtract 1 from 5, and there remains 4; to which bring down the next Point, it makes 4735 for the Resolvend. Then triple the 1, and it makes 3; and the Square of 1 is 1, and the Triple thereof is 3; which set one under another, in their Order, and added, makes 33 for the Divisor. Seek how often the Divisor in the Resolvend, and proceed as in the last Example.

```
.  .  .
5735339(179 Root.
1
———
4735 Resolvend.
———
  3  The Triple of the Quotient 1, the first Figure.
 3   The triple Square of the Quotient 1.
———
 33  The Divisor.
———
 343 The Cube of 7, the second Figure of the Root.
147  The Square of 7, multipl. in the triple Quot. 3.
21   The triple Square of the Quot. multiplied by 7.
———
3913 The Subtrahend.
————
 822339 The new Resolvend.
————
     51 The Triple of the Quot. 17, the two first Fig.
   867  The triple Square of the Quotient 17.
————
   8721 Divisor.
————
    729 The Cube of 9, the last Figure of the Root.
  4131  The Squ. of 9, multip. by the triple Quot. 51.
 7803   The triple Square of the Quotient 867 by 9.
————
 822339 The Subtrahend.
————
 ....
```

In

In this Example, 33, the first Divisor, seems to be contained more than seven times in 4735, the Resolvend; but if you work with 9, or 8, you will find that the Subtrahend will be greater than the Resolvend.

Some more Examples for Practice.

32461759(319	Root.
27	
5461	Resolvend.
9	The Triple of 3.
27	The triple Square of 3.
279	The Divisor.
1	The Cube of 1, the second Figure.
9	The triple Quotient, by the Square of 1.
27	The triple Square, multipied by 1, the 2d Fig.
2791	The Subtrahend.
2670759	A new Resolvend.
93	The Triple of 31.
2883	The triple Square of 31.
28923	The Divisor.
729	The Cube of 9, the last Figure.
7533	The Square of 9, by 93 the triple Quotient.
25947	The triple Square 2883 by 9.
2670759	The Subtrahend.

84604519(439	Root.
64	
20604	Resolvend.
12	Triple of 4.
48	Triple Square of 4.
492	Divisor.
27	Cube of 3.
108	Square of 3, by the triple Quotient.
144	Triple Square by 3.
15507	Subtrahend.
5097519	Resolvend.
129	Triple of 43.
5547	Triple Square of 43.
55599	Divisor.
729	Cube of 9.
10449	Square of 9 by 129.
49923	Triple Square by 9.
4097519	Subtrahend.
....	

259697989(638	
216	
43697	Resolvend.
18	Triple of 6.
108	Triple Square of 6.
1098	Divisor.
27	Cube of 3, the 2d Figure.
162	Triple Square of 3 by 18.
324	Triple Square 108 by 3.
34047	Subtrahend.
9650989	Resolvend.
189	Triple of 63.
11907	Triple Square of 63.
119259	Divisor.
512	Cube of 8.
12096	Square of 8 by 189.
95256	Triple Square 11907 by 8.
9647072	Subtrahend.
3917	Remainder.

```
 .  .  .
25917056(295.9
 8
-------
17917 Resolvend.
-------
   6  Triple of 2.
  12  Triple Square of 2.
-------
 126  Divisor.
-------
  729  Cube of 9, the 2d Figure.
 486   Square of 9 by 6.
108    Triple Square by 9.
-------
16389 Subtrahend.
-------
  1528056 Resolvend.
       87 Triple of 29.
     2523 Triple Square of 29.
---------
    25317 Divisor.
---------
      125 Cube of 5, the 3d Fig.
     2175 Square of 5 by 87.
    12615 Triple Square by 5.
---------
  1283375 Subtrahend.
    244681000 Resolvend.
    ---------
          885 Triple of 295.
       261075 Triple Square of 295.
      -------
      2611635 Divisor.
      -------
          729 Cube of 9, the last Figure.
        71685 Square of 9 by 885.
      2349675 Triple Square by 9.
    ---------
    235685079 Subtrahend.
    ---------
      8995921 Remainder.
```

In this Example I annex 3 Cyphers to the Remainder, which makes the 3d Resolvend; by which means I bring one Place of Decimals. And so you may proceed to more decimal Places at Pleasure, by annexing three Cyphers to the next Remainder, and carrying on the Work as before.

```
 .  .  .  .
22069810125(2805
8
————
14069 Reſolvend.
————
    6 Triple of 2.
   12 Triple Square of 2.
————
  126 Diviſor.
————
  512 Cube of 8.
 384  Square of 8 by 6.
 96   Triple Square by 8.
————
13952 Subtrahend.
    117810125 New Reſolvend.
————
         84 Triple of 28.
       2352 Triple Square of 28.
————
      23604 Diviſor.
————
        840 Triple of 280.
     235200 Triple Square of 280.
————
    2352840 New Diviſor.
————
        125 Cube of 5.
      21000 Square of 5 by 840.
    1176000 Triple Square by 5.
————
  117810125 Subtrahend.
————
.........
```

In this Example 13952, being ſubtracted from the Reſolvend 14069, the Remainder is 117; to which bring down 810, the 3d Point, and it makes 117810, for a new Reſolvend; and the next Diviſor is 23604, which you cannot have in the ſaid Reſolvend (the Unit's Place being rejected); ſo you muſt put 0 in the Quotient, and ſeek a new Diviſor (after you have brought down your laſt Point to the Reſolvend); which new Diviſor is 2352840; which you will find to be contained 5 times. So proceed to finiſh the reſt of the Work.

93759·575070(45·42	
64	
29759	Resolvend.
12	Triple of 4, the first Figure.
48	Triple Square of 4.
492	Divisor.
125	Cube of 5, the 2d Figure.
300	Square of 5 by 12, the triple Quotient.
240	Triple Square by 5.
27125	Subtrahend.
2634575	Resolvend.
135	Triple of 45.
6075	Triple Square of 45.
60885	Divisor.
64	Cube of 4.
2160	Square of 4 by 135.
24300	Triple Square by 4.
2451664	Subtrahend.
182911070	Resolvend.
1362	Triple of 45.4.
618348	Triple Square of 45.4.
6184842	Divisor.
8	Cube of 2.
5448	Square of 2 by 1362.
236696	Triple Square of 2 by 2.
123724088	Subtrahend.
59186982	Remainder.

In extracting the Cube Root of a mixed Number, always obſerve to make the decimal Part conſiſt of either three, ſix, nine, &c. Places, that is, always to conſiſt of even Points, as in the laſt Example, where the decimal Places were five; to which I annexed a Cypher to make up ſix, and ſo I proceed to point it; and by that means I have a Point fall upon the Unit's Place of whole Numbers, which you muſt always obſerve.

To extract the Cube Root out of a Fraction.

This is the ſame to do as in whole Numbers, obſerve but the foregoing Directions for the true pointing thereof; for, as was before directed, the Decimal muſt always conſiſt of three, ſix, nine, &c. Places; and if it be not ſo, it muſt be made ſo, by annexing Cyphers, as is above ſaid.

If the Cube Root of a Vulgar Fraction be required, you muſt firſt reduce it to a Decimal, and then extract the Root thereof.

Examples of each follow.

Example 1. Let the Cube Root of .401719179 be required.

```
 .  .  .
.401719179(.737 Root.
 343
 ---------
  58719 Resolvend.
 ---------
     21   Triple of 7.
    147   Triple Square of 7.
 ---------
   1491   Divisor.
 ---------
     27 Cube of 3.
    189 Square of 3 by 21.
   441  Triple Square by 3.
 ---------
  46017 Subtrahend.
 -----------
  12702179 Resolvend.
 -----------
       219 Triple of 73.
     15987 Triple Square of 73.
 -----------
    160089 Divisor.
 -----------
       343 Cube of 7.
     10731 Square of 7 by 219.
    111909 Triple Square by 7.
 -----------
  11298553 Subtrahend.
 -----------
   14c6236 Remainder.
```

Example 2. Let the Cube Root of .0001416 be required.

```
  .   .   .
.000141600(.052 Root.
    125
  -------
    16600 Resolvend.
  -------
       15  Triple of 5.
       75  Triple Square of 5.
     ----
      765  Divisor.
     ----
        8  Cube of 2.
       60  Square of 2 by 15.
      150  Triple Square by 2.
    -----
    15608  Subtrahend.
    -----
      992  Remainder.
```

Example 3. Let $\frac{2}{276}$ be a Vulgar Fraction, whose Cube Root is required.

By the first Rule of Chapter II. reduce the Vulgar Fraction to a Decimal.

```
276)5.000000000( 018115942
         ........
       ----------
        2240
          320
           440
           1640
            2600
             1160
               560
             -----
                 8
```

.018115942(.262	Root.
8	
10115	Resolvend.
6	Triple of 2.
12	Triple Square of 2.
126	Divisor.
216	Cube of 6.
216	Sq. of 6 by the Triple of 2.
72	Triple Square by 6.
9576	Subtrahend.
539942	Resolvend.
78	Triple of 26.
2028	Triple Square of 26.
20358	Divisor.
8	Cube of 2.
312	Square of 2 by 78.
4056	Triple Squ. 2028 by 2.
408728	Subtrahend.
131214	Remainder.

You may prove the Truth of the Work, by cubing the Root found, as was shewed in the first Example; and if any thing remains, add it to the said Cube, and the Sum will be the given Number, if the Work is rightly performed.

I will ſhew the Proof of the fifth Example (*Page* 50), the given Number being 259697989, whoſe Root is 638, it being a ſurd Number, there remains 3917.

	638
	638
	5104
	1914
	3828
The Square	407044
	638
	3256352
	1221132
	2442264
The Cube	259694072
The Remainder add	3917
Proof equal to the given Numb.	259697989

CHAP. IX.

Multiplication of Feet, Inches, and Parts.

IN the multiplying of Feet, Inches, *&c.* I ſhall endeavour to lay down ſuch eaſy and familiar Rules, as may eaſily be underſtood by the meaneſt Capacity.

Example

Example 1. Let 7 Feet 9 Inches be multiplied by 3 Feet 6 Inches.

F.	I.	
7	9	
3	6	
23	3	*Pts.*
3	10	6
27	1	6

First, Multiply 9 Inches by 3, saying, 3 times 9 is 27 Inches, which make 2 Feet 3 Inches; set down 3 under Inches, and carry 2 to the Feet, saying, 3 times 7 is 21, and 2 that I carry make 23; set down 23 under the Feet.

Then begin with 6 Inches, saying, 6 times 9 is 54 Parts, which is 4 Inches and 6 Parts; set down 6 Parts, and carry 4, saying, 6 times 7 is 42, and 4 that I carry is 46 Inches, which is 3 Feet 10 Inches; which set down, and add all up together, and the Product is 27 Feet 1 Inch 6 Parts.

Example 2. Let 75 Feet 7 Inches be multiplied by 9 Feet 8 Inches.

F.	I.	
75	7	
9	8	
680	3	
50	4	8
730	7	8

First, Multiply by 9 Feet, saying, 9 times 7 is 63, which is 5 Feet 3 Inches; set down 3, and carry 5, saying, 9 times 5 is 45, and 5 I carry is 50; set down 0, and carry 5, saying, 9 times 7 is 63, and 5

is 68; ſet down 68, and proceed to multiply by 8 Inches, ſaying, 8 times 7 is 56; the Twelves in 56 are four times, and 8 remains; ſet 8 a Place to the Right Hand, and carry 4: Then multiply 75 by 8, and the Product is 600, and 4 that I carry is 604, which divided by 12, the Quotient in 50 Feet, and 4 remains; ſet down 50 Feet 4 Inches, and add all up together, and you will find the Product 730 Feet 7 Inches 8 Parts.

I will repeat the laſt Example again, and ſhew another Way to work it, which, I think, is better, and more expeditious, when there are more Figures than one in the Feet; thus,

F.	I.	
75	7	
9	8	
680	3	
25	2	4
25	2	4
730	7	8

Multiply by 9 Feet, firſt, as above directed; then, inſtead of multiplying by 8 Inches, let the 8 Inches be parted into ſuch aliquot or even Parts of a Foot, as you find to be contained in that Figure; if you take ſuch Parts of the Multiplicand, and add them to the former Product, the Sum will give the Anſwer: Thus, 8 Inches may be parted into 4, and 4, becauſe 4 is the third Part of 12. So, if you take the third Part of 75 Feet 7 Inches, and ſet it down twice, and add all together, the Sum will be 730 Feet 7 Inches 8 Parts, the ſame as before; thus, ſay how often 3 in 7, which is twice; ſet down 2; then, becauſe twice 3 is 6, ſay, 6 out of 7, and there remains 1, for which you muſt add 10 to the 5, and it makes 15; then the Threes in 15 are 5 times; ſet down 5; and, becauſe 3 times 5 is 15, there is 0 remains. Then go to the

7 Inches, ſaying, the Threes in 7 are twice; ſet down 2 in the Inches; and becauſe twice 3 is but 6, take 6 out of 7, and there remains 1 Inch, which is 12 Parts; then the Threes in 12 are 4 times, and 0 remains. So the third Part of 75 Feet 7 Inches is 25 Feet 2 Inches 4 Parts; which ſet down again, and add all together, the Sum is 730 Feet 7 Inches 8 Parts; the ſame as before.

Example 3. Let 97 Feet 8 Inches be multiplied by 8 Feet 9 Inches.

F.	I.
97	8
8	9
781	4
48	10
24	5
854	7

Begin, firſt, to multiply by 8 Feet, ſaying, 8 times 8 is 64 Inches, that is, 5 Feet 4 Inches; ſet down 4 Inches, and carry 5, ſaying, 8 times 7 is 56, and 5 I carry is 61; ſet down 1, and carry 6, ſaying, 8 times 9 is 72, and 6 I carry is 78, which ſet down: Then, inſtead of multiplying by 9 Inches, take the aliquot Parts of 12 which 9 makes, which is 6 and 3; 6 Inches being half 12, and 3 the fourth Part; therefore take the half of 97 Feet 8 Inches, which is 48 Feet 10 Inches; and becauſe 3 is half 6, you may take the half of 48 Feet 10 Inches, which is 24 Feet 5 Inches; add all up together, and the Sum is 854 Feet 7 Inches. See the Work, as above.

Example 4. Let 75 Feet 9 Inches be multiplied by 17 Feet 7 Inches.

F.	I.	
75	9	
17	6	
525		
75		
25	3	P.
18	11	3
8	6	
4	3	
1331	11	3

In this Example, because there are more than 12 Feet in the Multiplier, therefore I first multiply the 75 by 17 Feet; then, because the aliquot Parts in 7 Inches are 4 and 3, that is, a third and a fourth, I take the third Part of 75 Feet 9 Inches, which is 25 Feet 3 Inches, and the fourth Part thereof is 18 Feet 11 Inches 3 Parts; then the aliquot Parts of 9 Inches are 6 and 3, that is, half and a fourth; therefore I take half 17 Feet, which is 8 Feet 6 Inches, and the fourth Part is 4 Feet 3 Inches (not meddling with the 7 Inches, because that was multiplied into the 9 before); then add all these together, and the Sum is 1331 Feet 11 Inches 3 Parts.

Example

Example 5. Let 87 Feet 5 Inches be multiplied by 35 Feet 8 Inches.

F.	I.	
87	5	
35	8	
435		
261		P.
29	1	8
29	1	8
11	8	0
2	11	0
3117	10	4

Work here as in the laſt Example. After you have multiplied the Feet, then take the aliquot Parts of 8 Inches, which are two Thirds; therefore take the third Part of 87 Feet 5 Inches, and ſet it down twice. Thus the third Part of 87 Feet 5 Inches is 29 Feet 1 Inch 8 Parts; ſet this down twice; then the aliquot Parts of 5 Inches are 4 and 1, that is, a third Part and a 12th Part; therefore take a third Part of 35, which is 11 Feet 8 Inches, and a 12th Part of 35 is 2 Feet 11 Inches; ſet all theſe one under another, and add them together, and the Sum is 3117 Feet 10 Inches 4 Parts.

Example 6. Let 259 Feet 2 Inches be multiplied by 48 Feet 11 Inches.

F.	I.	
259	2	
48	11	
2072		
1036		
129	7	P.
86	4	8
21	7	2
8	0	0
12677	6	10

 First,

Firſt, multiply the Feet; then take the aliquot Parts of 11, which will be 6, 4, and 1, that is, a half, a third, and a twelfth; therefore take the half of 297 Feet 2 Inches, which is 129 Feet 7 Inches, and a third Part is 86 Feet 4 Inches 8 Parts, and the twelfth Part of 259 Feet 2 Inches is 21 Feet 7 Inches 2 Parts; or (becauſe 1 is the fourth Part of 4), you may more readily take the fourth Part of 86 Feet 4 Inches 8 Parts, which is alſo 21 Feet 7 Inches 2 Parts; then 2 Inches are the ſixth of 12, take the ſixth of 48 Inches, which will be 8 Inches, which place under the Inches; then add all together, and the Sum is 12677 Feet 6 Inches 10 Parts. See the foregoing Work.

I ſhall ſet down only the Working of ſome few Examples in Feet and Inches, and then proceed to multiply Feet, Inches, and Parts, *&c.*

	F.	I.	P.
	179	3	
	38	10	
	1432		
	537		
	89	7	6
	59	9	0
	9	6	0
Product	6960	10	6

	F.	I.	P.
	246	7	
	46	4	
	1476		
	984		
	82	2	4
	15	4	0
	11	6	0
Product	11425	0	4

	F.	I.	P.
	246	7	
	36	9	
	1476		
	738		
	123	3	6
	61	7	9
	12	0	0
	9	0	0
	9061	11	3

	F.	I.	P.
	257	9	
	39	11	
	2313		
	771		
	128	10	6
	85	11	0
	21	5	9
	19	6	0
	9	9	0
Product	10288	6	3

Example

Example 11. Let 7 Feet 5 Inches 9 Parts be multiplied by 3 Feet 5 Inches 3 Parts.

F.	I.	P.		
7	5	9		
3	5	3		
22	5	3	S.	
3	1	4	9	T.
	1	10	5	3
25	8	6	2	3

In this Example, I first begin with 3 Feet, and thereby multiply 7 Feet 5 Inches 9 Parts: First, I say, 3 times 9 is 27 Parts, that is, 2 Inches and 3 Parts; set down 3 under the Parts, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Foot 5 Inches; set down 5 Inches, and carry 1, and say, 3 times 7 is 21, and 1 I carry is 22; set down 22 Feet: Then begin with 5 Inches, saying 5 times 9 is 45, which is 45 Seconds, which make 3 Parts and 9 Seconds; set down 9 Seconds a Place towards the Right Hand, and carry 3 Parts, saying, 5 times 5 is 25, and 3 I carry is 28, which is 2 Inches and 4 Parts; set down 4 Parts, and carry 2, saying, 5 times 7 is 35, and 2 I carry is 37, which is 3 Feet 1 Inch; set down 3 Feet 1 Inch, and begin to multiply by 3 Parts, saying, 3 times 9 is 27 Thirds, that is, 2 Seconds and 3 Thirds; set down 3 Thirds, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Part and 5 Seconds; set down 5 Seconds, and carry 1, saying, 3 times 7 is 21, and 1 I carry is 22, which is 1 Inch and 10 Parts, which set down, and add all up, and the Product is 25 Feet 8 Inches 6 Parts 2 Seconds 3 Thirds.

Note, That in multiplying Feet, Inches, and Parts, &c. if Feet be multiplied by Feet, the Product is Feet; and Feet multiplied by Inches, the Product is Inches;

and

and the twelfth Part is Feet; and Parts multiplied by Feet, the Product is Parts, and the twelfth Part thereof is Inches; Parts multiplied by Inches, the Product is Seconds, and the twelfth Part thereof is Parts; and Parts multiplied by Parts, the Product is Thirds, and the twelfth Part thereof is Seconds. So that if you begin to multiply Parts by Feet in the first Row, and Parts by Inches in the second Row, and Parts by Parts in the third Row, the first Figure in every Row will stand a Place more towards the Right Hand, as you may see in the last Example.

Example 12. Let 37 Feet 7 Inches 5 Parts be multiplied by 4 Feet 8 Inches 6 Parts.

F.	I.	P.		
37	7	5		
4	8	6		
150	5	8	S.	
12	6	5	8	
12	6	5	8	T.
1	6	9	8	6
177	1	5	0	6

First, I multiply by 4 Feet, saying, 4 times 5 is 20, which is 1 Inch 8 Parts; set down 8, and carry 1, saying, 4 times 7 is 28, and 1 I carry is 29, which is 2 Feet 5 Inches; set down 5 Inches, and carry 2, saying, 4 times 7 is 28, and 2 I carry is 30; set down 0, and carry 3, and say, 4 times 3 is 12, and 3 is 15; set down 15. Then I begin with 8 Inches; but, because the Feet in the Multiplicand are more than 12, it will be the best Way to work for the aliquot Parts of 8; so here I work for 4 Inches, and set that down twice, 4 being the third Part of 12; therefore take the third Part of 37 Feet 7 Inches 5 Parts, which is twelve Feet six Inches five Parts eight Seconds; set this down twice: Then begin with 6 Parts; but, instead of multiplying, take half 37 Feet 7 Inches

5 Parts (becauſe 6 is half 12), and ſet it a Place more to the Right Hand: Thus, the half of 37 Feet is 18, which I muſt count 18 Inches, becauſe the Multiplier is 6 Parts; ſo the half of 37 Feet 7 Inches 5 Parts, is one Foot ſix Inches nine Parts eight Seconds ſix Thirds; which ſet down, and add all up together, and the Sum is 177 Feet 1 Inch 5 Parts 0 Seconds 6 Thirds.

Example 13. Let 311 Feet 4 Inches 7 Parts be multiplied by 36 Feet 7 Inches 5 Parts.

F.	I.	P.	S.	T.
311	4	7		
36	7	5		
1866				
933				
103	9	6	4	
77	10	1	9	
8	7	9	6	4
2	1	11	4	7
12	0	0	0	0
1	0	0	0	0
	9	0	0	0
11402	2	4	11	11

In this Example, becauſe the Feet both in the Multiplier and Multiplicand are compound Numbers, I firſt multiply the Feet one by the other; then take the aliquot Parts of 7 Inches, which are 4 Inches and 3, that is, a third and a fourth Part; ſo take the third Part of 311 Feet 4 Inches 7 Parts, which is 103 Feet 9 Inches ſix Parts 4 Seconds, and the fourth Part is 77 Feet 10 Inches 1 Part 9 Seconds; ſet theſe down one under another, the Feet under the other Feet; then the aliquot Parts of 5 Parts are 4 and 1, that is, a third and twelfth Part; ſo the third Part of 311 Feet 4 Inches 7 Parts is 103 Feet 9 Inches 6 Parts 4 Seconds; but, becauſe the Multiplier is Parts, it muſt be ſet a Place

to

to the Right Hand, that is, the 103 muſt be Inches, which is 8 Feet 7 Inches; therefore I ſet down 8 Feet 7 Inches 9 Parts 6 Seconds 4 Thirds. Then, becauſe 1 Inch is a fourth Part of 4 Inches, therefore I take a fourth Part of 8 Feet 7 Inches 9 Parts 6 Seconds 4 Thirds, which is 2 Feet 1 Inch 11 Parts 4 Seconds 7 Thirds, which is the ſame as if I had taken a twelfth Part of 311 Feet 4 Inches 7 Parts. Then for 4 Inches in the Multiplicand, inſtead of multiplying 36 Feet by it, take a third Part, becauſe 4 Inches is the third Part of 12; ſo the third Part of 36 is 12 Feet, and the aliquot Parts of 7 Parts are 4 and 3, that is, a third and a fourth; ſo the third Part of 36 is 12, which now is 12 Inches, that is, 1 Foot, and the fourth Part is 9 Inches; add all theſe together, and the Sum will be 11402 Feet 2 Inches 4 Parts 11 Seconds 11 Thirds.

Example 14. Let 8 Feet 4 Inches 3 Parts 5 Seconds 6 Thirds be multiplied by 3 Feet 3 Inches 7 Parts 8 Seconds 2 Thirds.

	F.	I.	P.	S.	T.				
	8	4	3	5	6				
	3	3	7	8	2				
	25	0	10	4	6				
	2	1	0	10	4	6			
		1	10	6	0	2	6		
			5	6	0	3	8	0	
				1	4	8	6	11	0
Product	27	7	3	5	1	8	8	11	0

In this laſt Example there is no Difficulty, if you do but obſerve the former Directions, and ſet every Row a Place more to the Right Hand.

I ſhall only ſet down the Working of ſome few Examples more, and ſo conclude this Chapter.

F.	I.	P.		
321	7	3		
9	3	6		
2894	5	3	S.	
80	4	9	9	T.
13	4	9	7	6
2988	2	10	4	6

F.	I.	P.		
42	7	8		
7	3	6		
298	5	1		
10	7	11	S.	
1	9	3	10	
310	10	10	10	

F.	I.	P.		
124	7	9		
14	6	2		
496				
124			S.	
62	3	10	6	T.
1	8	9	3	6
7	0	0	0	0
1	2	0	0	0
	7	0	0	0
	2	6	0	0
1809	0	1	9	6

F.	I.	P.		
259	10	8		
18	5	4		
2072				
259			S.	
86	7	6	8	
21	7	10	8	T.
7	2	7	6	8
9	0	0	0	0
6	0	0	0	0
1	0	0	0	0
4793	6	0	10	8

F.	I.	P.		
267	7	10		
25	9	7		
1335				
534				
133	9	11	S.	
66	10	11	6	
11	1	9	11	T.
1	10	3	7	10
12	6	0	0	0
2	1	0	0	0
1	0	6	0	0
	8	1	0	0
6905	0	7	0	10

F.	I.	P.		
317	9	7		
37	5	9		
2219				
951			S.	
105	11	2	4	
26	5	9	7	T.
13	2	10	9	6
6	7	5	4	9
18	6	0	0	0
9	3	0	0	0
1	6	6	0	0
	3	1	0	0
11910	9	11	1	3

THE

The Complete Meaſurer, &c.

PART II.

CHAP. I.

Menſuration of SUPERFICIES.

Superficial Figures are all ſuch as have only Length and Breadth, not having any commenſurable Thickneſs.

§ I. *Of a* SQUARE.

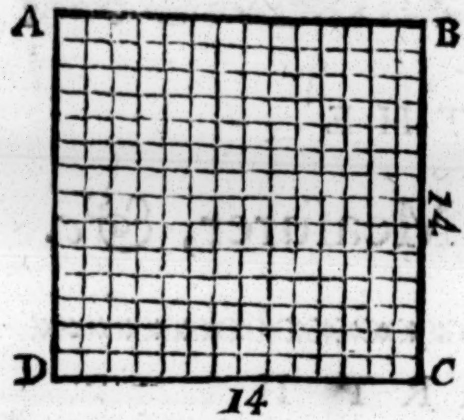
A SQUARE is a Geometrical Figure, having four equal Sides, and as many right (or ſquare) Angles. To find the ſuperficial Content thereof, this is

The RULE.

Multiply the Side into itſelf, and the Product is the Content.

Let

Let ABCD be a Geometrical Square given, each Side being 14 Feet, Yards, Poles, or other Meafure; multiply 14 by itfelf, and the Product is 169, which is the fuperficial Content.



$$\begin{array}{r} 14 \\ 14 \\ \hline 56 \\ 14 \\ \hline 196 \end{array}$$ Product.

By Scale and Compaffes.

Extend the Compaffes from 1, in the Line of Numbers, to 14; the fame Extent will reach from the fame Point, turned forward to 196.

Demonftration. Let each Side of the given Square be divided into 14 equal Parts, and Lines drawn from one another, croffing each other within the Square; fo fhall the whole great Square be divided into 196 little Squares, as you may fee in the Figure above, equal to the Number of Square Feet, Yards, Poles, or other Meafures, by which the Side was meafured.

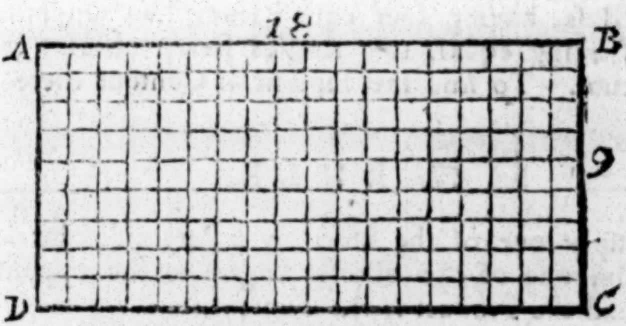
§ II.

§ II. *Of a* PARALLELOGRAM, *or* *Long Square.*

A Parallelogram is a Figure having four Sides, and as many Right Angles, the oppoſite Sides thereof being equal and parallel. To find the ſuperficial Content thereof, this is

The RULE.

Multiply the Length by the Breadth, and the Product is the ſuperficial Content.



Length — 18
Breadth — 9

Product —162

Let ABCD be a long Square, the Length thereof 18 Feet, and the Breadth 9 Feet; which multiplied together, the Product is 162, the ſuperficial Content thereof.

By Scale and Compaſſes.

Extend the Compaſſes in the Line of Numbers from 1 to 9, the ſame Extent will reach from 18 down to 162, the ſquare Feet.

Demonstration. If the Sides AB and CD be each divided into 18 equal Parts, representing 18 Feet; and the Lines AD and BC each divided into 9 equal Parts, and Lines drawn from Point to Point, crossing each other within the Figure; those Lines will make thereby so many little Squares as there are square Feet; *viz.* 162.

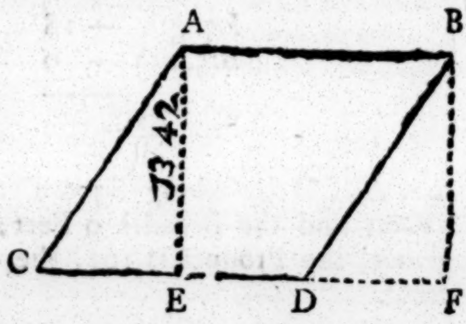
§ III. *Of a* RHOMBUS.

A Rhombus is a Figure representing a Quarry of Glass, having four equal Sides, the Opposites thereof being equal, two Angles being obtuse, and two acute. To find the superficial Content thereof, this is

The RULE.

Multiply one of the Sides by a Perpendicular let fall from one of the obtuse Angles to the opposite Side, and the Product is the Content.

Perpend.	13.42
The Side	15.5
	6710
	6710
	1342
Product	208.010



Let ABCD be a Rhombus given, whose Sides are each 15.5 Feet, and the Perpendicular EA is 13.42, which multiplied together, the Product is 208.010; which is the superficial Content of the Rhombus, that is, 208 Feet and one hundredth Part of a Foot.

By

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 13.42, that Extent will reach from 15.5, the ſame Way to 208 Feet the Content.

Demonſtration. Let CD be extended out to F, makeing DF equal to CE, and draw the Line BF; ſo ſhall the Triangle DBF be equal to the Triangle ACE: For DF and CE are equal, and BF is equal to AE, becauſe AB and CF are parallel. Therefore the Parallelogram ABEF is equal to the Rhombus ABCD.

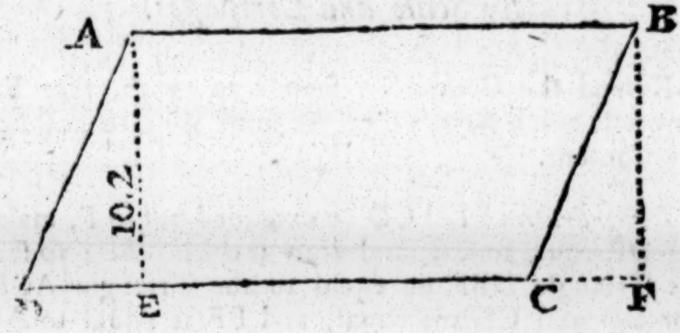
§ IV. *Of a* RHOMBOIDES.

A Rhomboides is a Figure having four Sides, the oppoſite whereof are equal and parallel; and alſo four Angles, the oppoſite whereof are equal. To find the ſuperficial Content thereof, this is

The RULE.

Multiply one of the longeſt Sides thereof by the Perpendicular let fall from one of the obtuſe Angles to one of the longeſt Sides, and the Product is the Content.

$$\begin{array}{r} 19.5 \\ 10.2 \\ \hline 390 \\ 195 \\ \hline 198.90 \end{array}$$



Let ABCD be a Rhomboides given, whoſe longeſt Sides, AB or CD, is 19.5 Feet, and the Perpendicular AE is 10.2; which multiplied together, the Product is 198.9, that is, 198 ſuperficial Feet and 9 tenth Parts, the Content.

Demonſtration. If DC be extended to F, making CF equal to DE, and a Line drawn from B to F; ſo will the Triangle CBF be equal to the Triangle ADE, and the Parallelogram AEFB be equal to the Rhomboides ABCD; which was to be proved.

§ V. *Of a* TRIANGLE.

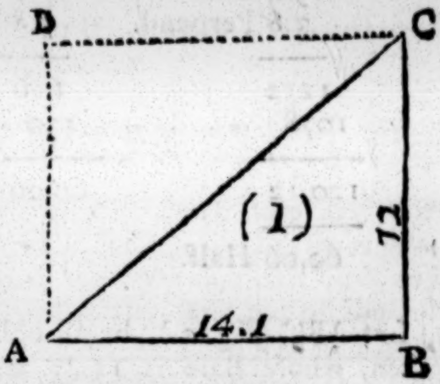
A Triangle is a Figure having three Sides and three Angles. Triangles are either right-angled or oblique-angled. Right-angled Triangles are ſuch as have one Right Angle. Oblique-angled Triangles are ſuch as have their Angles either acute or obtuſe. An obtuſe Angle is greater than a Right Angle, that is, it is more than 90 Degrees; and an acute Angle is leſs than a Right Angle. To find the ſuperficial Content thereof, this is

The

The RULE.

Let the Triangle be of what kind soever, multiply the Base by half the Perpendicular, or half the Base by the whole Perpendicular; or, multiply the whole Base by the whole Perpendicular, and take half the Product; any of these three Ways will give the Content.

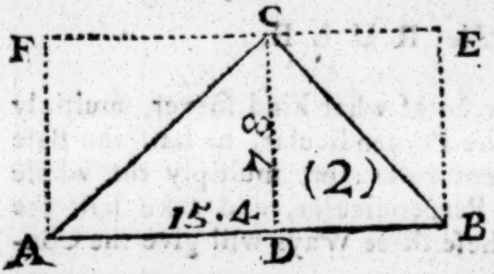
Let A B C be a Right-angled Triangle, whose Base is 14.1 Feet, and the Perpendicular 12 Feet. Multiply 14.1 by 6, half the Perpendicular, and the Product is 84.6 Feet, the Content. Or, multiply 14.1 by 12, the Product is 169.2; the half thereof is 84.6, the same as before.



14.1 Base.	14.1 Base.
6 Half Perpendicular.	12 Perpendicular.
84.6 Product.	169.2 Product.
	84.6 Half.

By Scale and Compasses.

Extend the Compasses from 2 to 14.1, that Extent will reach the same Way from 12 to 84.6 Feet, the Content.




```
15.4 Baſe.
 3.9 Half Perp.
————
1386
462
————
60.06 Product.
```

```
15.4 Baſe              7.7 Half Baſe.
 7.8 Perpend.          7.8 Perpendicular.
————                   ————
  1232                  616
 1078                  539
————                  ————
120.12                60.06 Product.
————
60.06 Half.
```

Let ABC (*Fig.* 2.), be an oblique-angled Triangle given, whoſe Baſe is 15.4, and the Perpendicular 7.8; if 15.4 be multiplied by 3.9 (half the Perpendicular), the Product will be 60.06 for the Area, or ſuperficial Content: Or, if the Perpendicular 7.8 be multiplied into half the Baſe 7.7, the Product will be 60.06 as before: Or, if 15.4, the Baſe, be multiplied by the whole Perpendicular 7.8, the Product will be 120.12, which is the double Area; the Half thereof is 60.06 Feet, as before. See the Work.

By Scale and Compaſſes.

Extend the Compaſſes from 2 to 15.4, that Extent will reach from 7.8 to 60 Feet, the Content.

Demonſtration. If AD (*Fig.* 1.) be drawn parallel to BC, and DC parallel to AB; the Triangle ADC ſhall be equal to the given Triangle ABC. Hence the Parallelogram ABCD is double to the given Triangle; therefore half the Area of the Parallelogram is

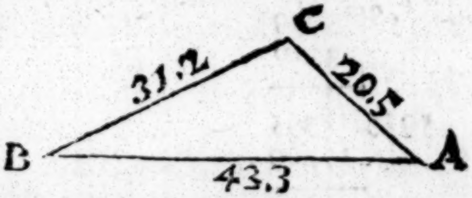
is the Area of the Triangle. In *Fig.* 2. the Parallelogram ABEF is alſo double to the Triangle ABC; for the Triangle ACF is equal to the Triangle ACD, and the Triangle BCE is equal to the Triangle BCD; therefore the Area of the Parallelogram is double to the Area of the given Triangle. Which was to be proved.

To find the Area of any plain Triangle, by having the three Sides given, without the Help of a Perpendicular.

The RULE.

Add the three Sides together, and take half that Sum: Then ſubtract each Side ſeverally from that half Sum. Which done, multiply that half Sum and the three Differences continually, and out of the laſt Product extract the Square Root; which Square Root ſhall be the Area of the Triangle ſought.

Example. Let ABC be a Triangle, whoſe three Sides are as followeth; *viz.* AB 43.3, AC 20.5, and BC 31.2, the Area is required.



Sides	43.3	4.2	Differences.
	31.2	16.3	
	20.5	27.0	
Sum	95.0		
Half	47.5		

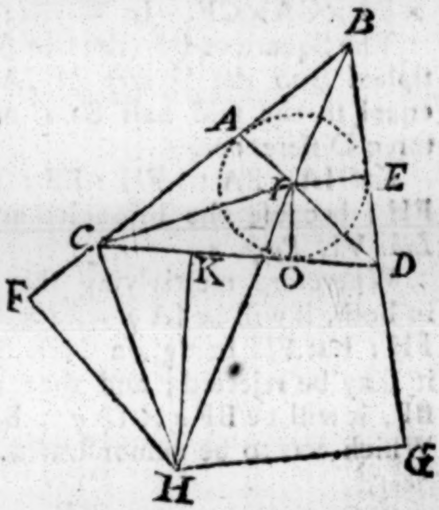
Area

Area 296.31

```
    47.5 The half Sum.
      27 Difference.
    ----
    3325
    950
    ------
  1282.5 Product.
    16.3 Difference.
   ------
   38475
  76950
 12825
 ---------
 20904.75 Product,
      4.2 Difference.
 ---------
  4180950
 8361900
 ---------
 .  .  .  .  .
 87799.9500(296.31
 4
 ----
 49)477
    441
 ----
 586)3699
     3516
 ------
 5923)18395
      17769
 ------
 59261)62600
       59261
 ------
       3339 Remains.
```

Demon-

Demonftration. In the Triangle BCD, I fay, if from the half Sum of the Sides, you fubtract each particular Side, and multiply the half Sum and the three Differences together continually, the Square Root of the Product fhall be the Area of the Triangle.



Firft, by the Lines BI, CI, and DI, bifect the three Angles, which Lines will all meet in the Point I; by which Lines the given Triangle is divided into three new Triangles, CBI, DCI, and BDI; the Perpendiculars of which new Triangles are the Lines AI, EI, and OI, being all equal to one another, becaufe the Point I is the Center of the infcribed Circle (by *Euclid, Lib.* IV. *Prop.* 4.): Wherefore to the Side BC join CF equal to DE, or DO; fo fhall BF be equal to half the Sum of the Sides; *viz.* $=\frac{1}{2}$ BC $+\frac{1}{2}$ BD $+\frac{1}{2}$ CD.

And BA=BF—CD; for CA=CO and OD=CF; therefore CD=AF; and AC = BF—BD, for BE= BA, and ED=CF; therefore BD=BA+CF, and CF=BF—BC.

Then make CK=CF, and draw the Perpendiculars FH, GH, and KH, and extend BI to H; becaufe the Angle FCK+FHK are equal to two Right Angles (for the Angles F and K are Right Angles) equal alfo to FCK+ACO (by *Euclid,* I. 13.), and the Angles ACO+AIO are equal to two Right Angles; therefore the Quadrangles FCKH and AIOC are alike; and the Triangles CFH and AIC are alfo fimilar. And the Triangles BAI and BFH are likewife fimilar.

From

From this Explanation, I ſay, the Square of the Area of the given Triangle will be $BFq \times IAq = BF \times BA \times CA \times CF$. In Words;

The Square of BF (the half Sum of the Sides) multiplied into the Square of IA ($=IF=IO$) will be equal to the ſaid half Sum multiplied into all the three Differences.

For IA : BA : : FH : BF; and IA : CF : : AC : FH; becauſe the Triangles are ſimilar. By *Euclid, Lib.* VI. *Prop.* 4.

Wherefore multiplying the Extremes and Means in both, it will be $IAq \times BF \times FH = BA \times CA \times CF \times FH$; but FH being on both Sides of the Equation, it may be rejected; and then multiply each Part by BF, it will be $BFq \times IAq = BF \times BA \times CA \times CF$. Which was to be demonſtrated.

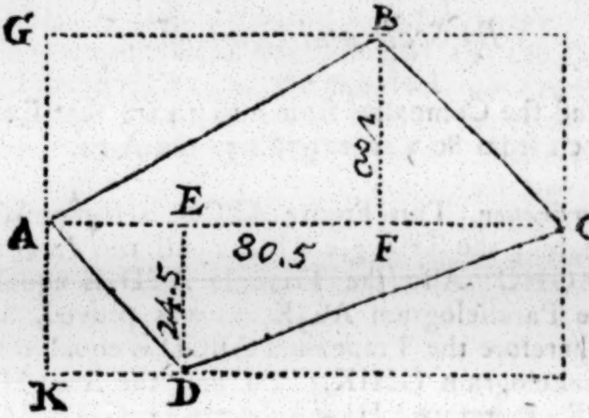
§ VI. *Of a* TRAPEZIUM.

A Trapezium is a Figure having 4 unequal Sides, and oblique Angles: To find the Area or ſuperficial Content thereof, this is

The RULE.

Add the two Perpendiculars together, and take half the Sum, and multiply that half Sum by the Diagonal, or multiply the whole Sum by half the Diagonal, the Product is the Area. Or you may find the Areas of the two Triangles, ABC and ACD (by Section V.), and add thoſe two Areas together, the Sum ſhall be the Area of the Trapezium.

BF=30.4



BF=	30.1
DE=	24.5
Sum	54.6
Half	27.3
AC=	80.5
	1365
	2184
Area	2197.65

Let ABCD be a Trapezium given, the Diagonal whereof is 80.5, and the Perpendicular BF 30.1, and the Perpendicular DE 24.5; theſe two added together, the Sum is 54.6, the Half thereof is 27.3, which multiplied by the Diagonal 80.5, the Product is 2197.65, which is the Area of the Trapezium; or if 40.25, half the Diagonal, be multiplied by 54.6, the whole Sum of the Perpendiculars, the Product is 2197.65, the ſame as before.

By

By Scale and Compaffes.

Extend the Compaffes from 2 to 54.6; that Extent will reach from 80.5 to 21976.65, the Area.

Demonftration. This Figure ABCD is compofed of two Angles; the Triangle ABC is half the Parallelogram AGHC: Alfo the Triangle ACD is equal to half the Parallelogram ACIK, as was proved, Sect. V. Wherefore the Trapezium ABCD is equal to half the Parallelogram GHIK. To find the Area HI$=$BF$+$DE; therefore $\frac{1}{2}$ HI$\times$AC ($=$KI$=$GH) $=$Area of the Trapezium. Which was to be proved.

§ VII. *Of* IRREGULAR FIGURES.

IRregular Figures are all fuch as have more Sides than four, and the Sides and Angles unequal. All fuch Figures may be divided into as many Triangles as there are Sides, wanting two. To find the Area of fuch Figures, they muft be divided into Trapeziums and Triangles, by Lines drawn from one Angle to another; and fo find the Areas of the Trapeziums and Triangles feverally, and then add all the Areas together; fo will you have the Area of the whole Figure.

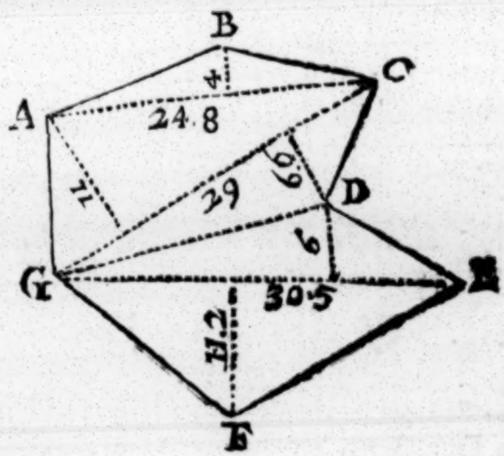
Let ABCDEFG be an irregular Figure given to be meafured; firft, draw the Lines AC and GD, and thereby divide the given Figure into two Trapeziums, ACGD and GDEF, and the Triangle ABC; of all which I find the Areas feverally.

Firft, I multiply the Bafe AC by half the Perpendicular, and the Product is 49.6, the Area of the Triangle ABC.

7 Then

Then for the Trapezium ACGD, the two Perpendiculars, 11 and 6.6, added together, make 17.6; the half thereof is 8.8, multiplied by 29, the Diagonal; the Product is 255.2, the Area of that Trapezium.

And for the Trapezium GDEF, the two Perpendiculars, 11.2 and 6, added together, make 17.2; the half thereof is 8.6; which multiplied by 30.5, the Diagonal, the Product is 262.3, the Area thereof. All thefe Areas added together, make 567.1, and fo much is the Area of the whole irregular Figure. See the Work.



24.8 Bafe AC.
2 Half Perpendicular.
———
49.6 Area of ABC.

11 Perpendicular.
6.6
———
17.6 Sum.
———
8.8 Half.
29 Diagonal CG.
———
792
176
———
255.2 Area of ACGD

11.2 } Perpendiculars.	30.5	
6 }	8.6	
17.2 Sum.	1830	
	2440	
8.6 Half Sum.		
	262.30	Area of GDEF.
	255.2	Area of ACGD.
	49.6	Area of ABC.
	567.1	Sum of the Areas.

This Figure being composed of Triangles and Trapeziums, and those Figures being sufficiently demonstrated in the Vth and VIth Sections aforegoing, it will be needless to mention any thing of the Demonstration thereof in this Place.

§ VIII. *Of* Regular Polygons.

Regular Polygons are all such Figures as have more than four Sides, all the Sides and Angles thereof being equal. Polygons are denominated from the ber of their Sides and Angles.

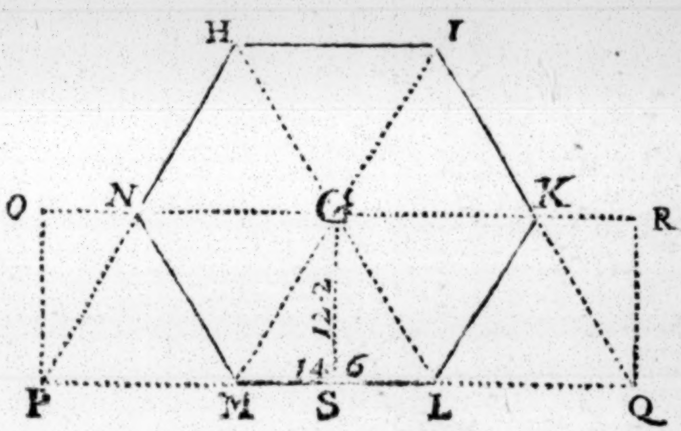
If the Figure consists of	5	Equal Sides and Angles, it is called a Regular	Pentagon.
	6		Hexagon.
	7		Heptagon.
	8		Octagon.
	9		Enneagon.
	10		Decagon.
	11		Endecagon.
	12		Dodecagon.

To

To find the Area or ſuperficial Content of any regular Polygon, this is

The RULE.

Multiply the whole Perimeter, or Sum of the Sides, by half the Perpendicular let fall from the Center to the Middle of one of the Sides; or multiply the half Perimeter by the whole Perpendicular, and the Product is the Area.



14 6
3

43 8 Half Sum of the Sides.

12.64 The Perpendicular.
43 8 Half Sum.

10112
3792
5056

553.632 Area.

14 6
6

87.6 Sum of the Sides.
6.32 Half Perpend.

1752
2628
5256

553 632 Area.

Let HIKLMN be a Regular Hexagon, each Side thereof being 14.6, the Sum of all the Sides is 87.6, the half Sum thereof is 43.8, which multiplied by the Perpendicular GS 12.64, the Product is 553.632: Or, if 87.6, the whole Sum of the Sides, be multiplied by half the Perpendicular 6.32, the Product is 553.632, the ſame as before, which is the Area of the given Hexagon.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 12.2, that Extent will reach from 43.8, the ſame Way to 553.632: Or, extend from 2 to 12.2, that Extent will reach from 87.6 to 553.632.

Demonſtration. Every regular Polygon is equal to the Parallelogram, or long Square, whoſe Length is equal to half the Sum of the Sides, and Breadth equal to the Perpendicular of the Polygon, as appears by the foregoing Figure; for the Hexagon HIKLMN is made up of ſix equilateral Triangles: And the Parallelogram OPQR is alſo compoſed of ſix equilateral Triangles, that is, five whole ones, and two Halves; therefore the Parallelogram is equal to the Hexagon.

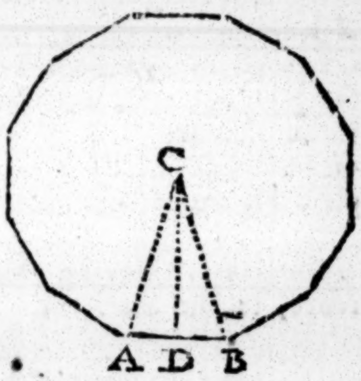
A Table *for the more ready finding the Area of a Polygon.*

Number of Sides.	Names.	Multipliers.
3	Trigon	.433013
4	Tetragon	1.000000
5	Pentagon	1.720477
6	Hexagon	2.598076
7	Heptagon	3.633959
8	Octagon	4.828427
9	Enneagon	6.181827
10	Decagon	7.694209
11	Endecagon	8.514250
12	Dodecagon	9.330125

Multiply the Square of the Side by the Tabular Number, and the Product is the Area of the Polygon.

How to find theſe Tabular Numbers.

Theſe Numbers are found by Trigonometry, thus: Find the Angle at the Center of the Polygon by dividing 360 Degrees by the Number of Sides of the Polygon.



Example. Suppoſe each Side of the Dodecagon annexed by 1, and the Area be required.

Divide 360 by 12 (the Number of Sides), and the Quotient is 30 Degrees for the Angle ACB; the Half thereof is 15, the Angle DCB, whoſe Complement to 90 Degrees is 75 Degrees, the Angle CBD: Then ſay,

As s, DCB 15 Degrees is	Co-ar. 0.587004
to .5 the Half-ſide DB. Log.	1 698970
ſo is s, CBD 73 Degrees,	9.984944
to the Perpendicular CD 1.866025	0.270918

Then 1 866025 multiplied by .5 (the Half-ſide) the Product is 9 330125 the Area of the Dodecagon required.

§ IX. *Of a* CIRCLE.

A Circle is a plain Figure, contained under one Line, which is called a Circumference, unto which all Lines drawn from a Point in the Middle of the Figure, called the Center, and falling upon the Circumference thereof, are all equal the one to the other. The Circle contains more Space than any plain Figure of equal Compaſs.

Problem 1. Having the Diameter and Circumference, to find the Area.

The RULE.

Every Circle is equal to a Parallelogram, whoſe Length is equal to half the Circumference, and the Breadth equal to half the Diameter; therefore multiply

ply half the Circumference by half the Diameter, and the Product is the Area of the Circle.

35.5 Half Circumf.
11.3 Half Diameter.

1065
355
355

401.15 Area.

Thus, if the Diameter of a Circle (that is, the Line drawn croſs the Circle through the Center) be 22.6; and if the Circumference be 71, the Half of 71 is 35.5, and the Half of 22.6 is 11.3; which multiplied together, the Product is 401.15, which is the Area of the Circle.

Demonſtration. Every Circle may be conceived to be a Polygon of an infinite Number of Sides, and the Semidiameter muſt be equal to the Perpendicular of ſuch a Polygon, and the Circumference of the Circle equal to the Periphery of the Polygon; therefore half the Circumference, multiplied by half the Diameter, gives the Area as aforeſaid.

Or (with *F. Ignat. Gaſton Pardies*), " Every Circle " is equal to a Rectangle Triangle, one of whoſe " Legs is the Radius, and the other a Right Line " equal to the Circumference of the Circle: For " ſuch a Triangle will be greater than any Polygon " inſcribed, and leſs than any Polygon circumſcribed, " by the 24th, 25th, 26th, and 27th Articles of the " fourth Book of his Elements of Geometry; and " therefore muſt be equal to the Circle.

" For

" For (ſays he) ſhould it be greater than the Circle, " be the Exceſs as little as it will, a Polygon may be " circumſcribed, whoſe Difference from the Circle, " ſhall be yet leſs than the Difference between that Cir- " cle and the Rectangle Triangle; and that that Poly- " gon will be leſs than the Triangle, is abſurd; and " if it be ſaid, that this rectangled Triangle is leſs " than the Circle, an inſcribed Polygon may be made, " which ſhall be greater than that Triangle; which " is impoſſible.

" This cannot but be admitted as a Principle, That " if two determinate Quantities, A and B, are ſuch " that if every imaginable Quantity, which is greater " or leſs than A, is alſo greater or leſs than B, theſe " two Quantities A and B muſt be equal.

" And this Principle being granted, which is in a " manner ſelf-evident, it may directly be proved, that " the Triangle (before mentioned) is equal to the " Circle; becauſe every imaginable inſcribed Figure, " which is leſs than the Circle, is alſo leſs than the " Triangle; and every circumſcribed Figure, greater " than the Circle, is alſo greater than the Triangle."

Problem 2. Having the Diameter of a Circle, to find the Circumference.

As 7 to 22, ſo is the Diameter to the Circumfe- rence.

Or, as 113 to 355, ſo is the Diameter to the Cir- cumference.

Or, as 1 to 3.141593, ſo is the Diameter to the Circumference.

Let the Diameter (as in the former Circle) be 22.6, this multiplied by 22, and the Product is 497.2; which, divided by 7, gives 71.028 for the Circum- ference.

ference. Or (by the ſecond Proportion) if 22.6 be multiplied by 355, the Product will be 8023; this divided by 11.3, the Quotient is 71, the Circumference. Or (by the third Proportion) if 22.6 be multiplied into 3.141593, the Product is 71.0000018 the Circumference; which two laſt Proportions are the moſt exact.

```
   22.6                    355
     22                   22.6
  -----                  -----
    452                   2130
   452                    710
  -----                  710
7)497.2(71.028          -------
                     113)8023.01(71
                          791
 3.141593                 ---
     22.6                 113
 --------                 113
 18849558                 ---
 6283186                  ...
6283186
---------
71.0000018
```

By Scale and Compaſſes.

Extend the Compaſſes from 7 to 22, or from 113 to 355, or from 1 to 3.14159; that Extent will reach from 22.6 to 71.

The Proportion of the Diameter of a Circle to the Circumference was never yet exactly found, notwithſtanding many eminent learned Men have laboured very far therein; among which the excellent *Van Culen* hath hitherto outdone all, in his having calculated the ſaid Proportion to 36 Places of Decimals, which are engraven upon his Tomb-ſtone in St. *Peter*'s Church in *Leyden*; which Numbers are theſe:

Diameter.

Diameter.

1.00000.00000.00000.00000.00000.00000.00000.

Circumference.

3 14159 26535.89793.23846.26433 83279.50288.

Of which large Number, theſe ſix Places, 3 14159, anſwering to the Diameter 1 00000, may be ſufficient; of the three Proportions, as 7 to 22, 113 to 355, and 1 to 3.14159, I ſhall leave my Reader to uſe which of them he pleaſes, but ſhall commend the laſt two as moſt exact, tho' the firſt be moſt in Uſe: But in the following Work I ſhall uſe ſometimes one of them, and ſometimes another; but for the moſt part that of *Van Culen*, as being moſt exact.

Problem 3. Having the Circumference of a Circle, to find the Diameter.

As 1 is to .318309, ſo is the Circumference to the Diameter.

Or, as 355 to 113, ſo is the Circumference to the Diameter.

Or, as 22 to 7, ſo is the Circumference to the Diameter.

Let the Circumference be 71 (as in the former Circle), if .318309 be multiplied by 71 (as by the firſt Proportion), the Product will be 22.599939 for the Diameter. Or, by the ſecond Proportion, 113 multiplied by 71, the Product is 8023; which divided by 355, the Quotient will be 22.6 the Diameter. Or, by the third Proportion, 71 multiplied by 7, the Product is 497; this divided by 22, the Quotient is 22.5909, the Diameter.

.318309		
71	113	71
	71	7
318309		
2228163	113	22)497(22.59
	791	44
22.599939		
	355)8023(22.6	57
	710	44
	923	130
	710	110
	2130	200
	2130	198
	...	2

Thus, by both the firſt Proportions, the Diameter is 22.6, but by the laſt it falls ſomething ſhort.

By Scale and Compaſſes.

Extend the Compaſſes from 3.14159 to 1, that Extent will reach from 71 to 22.6, which is the Diameter ſought.

Or, you may extend from 1 to .318309.

Or from 22 to 7.

Or from 355 to 113; the ſame will reach from 71 to 22.6, as before.

Note, That if the Circumference be 1, the Diameter will be .318309.

Problem 4. Having the Diameter of a Circle, to find the Area.

All Circles are in Proportion one to another, as are the Squares of their Diameters (by *Euclid*, 12.2). Now, the Area of a Circle, whoſe Diameter is 1, will be .785398, according to *Van Culen*'s Proportion before;

before-mentioned; but for Practice .7854 will be sufficient: Therefore,

As 1 (the Square of the Diameter 1) is to .7854, so is 510.76 (the Square of 22.6, the Diameter of the given Circle) to 401.15 (the Area of the given Circle): But,

According to *Metius*'s Proportion,

As 452 : 355 : : 510.76 : 401.15, the same as before.

But, if you use *Archimedes*'s Proportion, say,

As 14 : 11 : : 510.76 : 401.31; which Area is greater than by the two former Proportions; though in small Circles this is near enough the Truth. See the Working of all these.

```
    22.6 Diameter of the former Circle.
    22.6
   -----
    1356
    452
   452
  ------
  510.76 the Square of the said Diameter.

As 1 : .7854 : : 51076
          .7854
        -------
         204304
        255380
       408608
      357532
    -----------
    401.150904 the Area.
```

By

By Scale and Compasses.

The Extent from 1 to 22.6, being twice turned over from .7854, will fall at the last upon 401.15, the Area.

```
113          As 452 : 355 : : 510.76
  4                              355
----                          ------
452                           255380
                             255380
                            153228
                          ----------
                      452)181319.80(401.15
                          1808
                          ----
                             519
                             452
                             ---
                              678
                              452
                              ---
                              2260
                              2260
                              ----
                              ....
```

```
As 14 : 11 : : 510.76
                   11
              ---------
         14)5618 36(401 31
            56
            ---
             18
             14
             ---
              43
              42
              ---
               16
               14
               ---
                2
```

Problem 5. Having the Circumference of a Circle, to find the Area.

Becauſe the Diameters of Circles are proportional to their Circumferences; that is, as the Diameter of one Circle is to its Circumference; ſo is the Diameter of another Circle to its Circumference: Therefore the Areas of Circles are to one another, as the Squares of the Circumferences. And if the Circumference of a Circle be 1, the Area of that Circle will be .07958; then the Square of 1 is 1, and the Square of 71 (the Circumference of the former Circle) is 5041. Theref re it will be,

```
       Sq. Cir.  Area.  Sq. Circumf.
       As  1 : .07958 : : 5041
                 5041
              ----------
                  7958
22               31832
 4              397900
--             --------
88             401.16278
```

Or

Or thus :

As 88 : 7 : : 5041
7

88)35287(400.98 Area.
352

870
792

355   780
4   704

1420   76

Or, As 1420 : 113 : : 5041 : 401.15 Area.

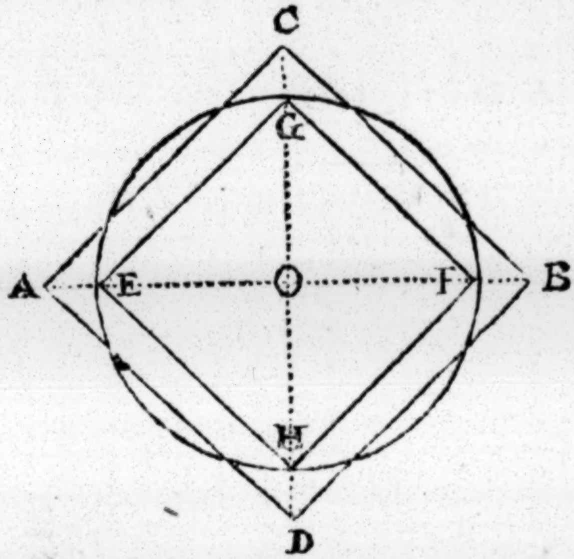
Problem 6. By having the Diameter, to find the Side of a Square that is equal in Area to that Circle.

If the Diameter of a Circle be 1, the Side of a Square equal thereunto will be .8862. Therefore,

As 1 : .8862 : : 22.6 (the Diameter)
22.6

53172
17724
17724

To 20.02812 the Side of the Square AC.



Let the Diameter of the Circle EF or GH, be 22.6 (as before), to find the Side of the Square AC, AD, *&c.* If .8862 be multiplied by 22.6, the Product is 20.02812, which is the Side of a Square, equal in Area to the Circle given; for if 20.02812 be multiplied Square-wife, that is, by itfelf, it will produce 401.1255907344, which is nearly equal to the Area found in the laft Problem.

You may find the Side of the Square equal, by extracting the Square Root out of the Area of the given Circle.

```
    401.15(20.0287295 Side of a Square.
    4
    ----------
4002)01.1500
        8004
    ----------
 40048)349600
  .... 320384
       ------
        29216
        28034
       ------
         1182
          801
       ------
          381
          360
       ------
           21
           20
           --
            1
```

N. B. By this Method of extracting the Square Root of the Area, you may find the Side of a Square equal to any plain Figure, regular or irregular.

Problem 7. By having the Circumference, to find the Side of the Square equal.

If the Circumference of a Circle be 1, the Side of the Square equal will be .2821. Therefore,

```
As 1 : .2821 : : 71 (the Circumference)
           71
       -------
         2821
       19747
       -------
       20.0291 the Side of the Square.
```

Problem 8. Having the Diameter, to find the Side of a Square, which may be inſcribed in that Circle.

If the Diameter of a Circle be 1, the Side of the Square inſcribed will be .7071. Therefore,

As 1 : 7071 : : 22 6
22.6

42426
14142
14142

To 15.98046 the Side EG inſcribed.

Or, if you ſquare the Semidiameter and double that Square, the Square Root of the doubled Square will be the Side of the Square inſcribed. For (by *Euclid*, 1.47), the Square of the Hypothenuſe EG is equal to the Sum of the other two Legs, EO and OG.

11.3 Semidiameter.
11.3

339
113
113

127.69 the Square of EO, which double, be-
2 [cauſe EO=OG.

255.38(15.98 Root, which is the Side of the Sq.
1

25)155
125

309)3038
2781

3188)25700
25504

196

Problem

Problem 9. Having the Circumference, to find the Side of a Square which may be inſcribed.

If the Circumference be 1, the Side of the Square inſcribed will be .2251. Therefore,

$$\begin{array}{r} \text{As } 1 : .2251 :: 71 \\ 71 \\ \hline 2251 \\ 15757 \\ \hline 15.9821 \text{ the Side of the Sq. EG.} \end{array}$$

Becauſe that in each of the four laſt Problems, *viz.* the 6th, 7th, 8th, and 9th, there is a Proportion laid down, it will be eaſy to work them with Scale and Compaſſes; for if you extend the Compaſſes from the firſt to the ſecond, that Extent will reach from the third to the fourth: As in the laſt Problem, where the Proportion is as 1 to .2251, ſo is 71 to the Side of the Square 15.9821. Here extend the Compaſſes from 1 to .2251; that Extent will reach from 71 to 15.98; and ſo of the reſt. But the fifth muſt be wrought like the fourth, thus; extend the Compaſſes from 1 to 71; that Extent, turned over the ſame way from .07958, will fall, at laſt, upon 401.15.

Problem 10. Having the Area, to find the Diameter.

If the Area of a Circle be 1, the Square of the Diameter thereof is 1.2732. Therefore,

Area.

```
Area. Sq. Diam.   Area.
As 1 : 1.2732 : : 401.15.
           401.15
         --------
           63660
          12732
         12732
       509280
      -----------
      . .  . . .
      510.744180(22.599 the Diameter.
      4
    ------
    42)110
        84
    ------
   445)2674
       2225
       ----
  4509)44941
       40581
     ---------
 45189)436080
       406701
       -------
        29379
```

By Scale and Compasses.

Extend the Compasses from 1 to 1.2732; that Extent will reach from 401.15 to 510.74, *&c.* Then divide the Space between 1 and 510.74 into two equal Parts, and you'll find the middle Point at 22.6. Or you may divide the Space upon the Line of Numbers, between 401.15 and .7854, into two equal Parts, and one of those Parts will reach from 1 to 22.6, the Diameter sought.

Problem

Problem 11. Having the Area, to find the Circumference.

If the Area of a Circle be 1, the Square of the Circumference will be 12.56637. Therefore,

```
Ar. Sq. Circumf.    Area.
As 1 : 12.56637 : : 401.15
          401.15
        --------
        6283185
       1256637
      1256637
    50265480
   -------------
    . . . . . . . Circumf.
    5040.99932550(70.9990 Root.
    49
   ------
1409)14099
      12681
     ------
14189)141893
       127701
      --------
141989)1419225
        1277901
       ---------
1419989)14132450
         12779901
        ----------
          1352549
```

By Scale and Compasses.

Divide the Space between 401.15 and .07958, upon the Line, into two equal Parts; one of those Parts will reach from 1 to 71, the Circumference sought.

Problem

Problem 12. Having the Area, to find the Side of a Square infcribed.

If the Area of a Circle be 1, the Area of a Square infcribed within that Circle will be .6366. Therefore,

```
As 1 : 401.15 : : .6366
          .6366
         ------
         240690
        240690
       120345
      240690
      -------
      . . . . .
      255.372090(1598 Root, which is the Side of
      1                         [the Square fought.
      ---
   25)155
      125
     ----
  309)3037
      2781
      -----
 3188)25620
      25504
      -----
       11690
```

The fame Reafon may be given for the laft Proportion, that was given before for the Proportion of Circles to the Squares of their Diameters and Circumferences; for not only the Squares of the Diameters and Circumferences are in Proportion to the Circles they belong to, but alfo all Figures infcribed or circumfcribed, have the Squares of their like Sides proportioned to the Circles they are infcribed in, or circumfcribed about; and alfo to the Figures themfelves: The Square of any Side of one Figure is to the Area of that

that Figure, as the Square of the like Side of another ſimilar Figure is to the Area thereof; as you may find proved at large in *Euclid*, *Sturmius*, *Matheſis*, *Enucleata*, and other Authors; but will be too large to inſert in this Place.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 401.15, that Extent will reach from .6366 to 255.37; the half Space between that and 1 is at 15.98, the Side of the Square.

Problem 13. Having the Side of a Square, to find the Diameter of the circumſcribing Circle.

If the Side of a Square be 1, the Diameter of a Circle that will circumſcribe that Square, will be 1.4142. Therefore,

$$\begin{array}{r}
\text{As } 1 : 1.4142 :: 15.98 \\
15.98 \\
\hline
113136 \\
127278 \\
70710 \\
14142 \\
\hline
22.598916
\end{array}$$

22.598916 the Diameter ſought.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 1.4142, and that Extent will reach from 15 98 to 22.6, the Diameter ſought.

Problem 14. Having the Side of a Square, to find the Diameter of a Circle equal.

If the Side of a Square be 1, the Diameter of a Circle equal thereunto will be 1.128. Therefore,

Side Diam. Side of a Square.
As 1 : 1.128 : : 20.0291
1.128

1602328
400582
200291
200291

22.5928248 Diameter.

By Scale and Compasses.

Extend the Compasses from 1 to 1.128; that Extent will reach from 20.0291 (the Side of the Square given,) to 22.6, the Diameter of the Circle sought.

Problem 15. Having the Side of a Square, to find the Circumference of a circumscribing Circle.

If the Side of a Square be 1, the Circumference of a Circle that will encompass that Square will be 4.443. Therefore,

Side Sq. Circum. Side Sq.
As 1 : 4.443 : : 15.98
15.98

35544
39987
22215
4443

70.99914 the Circumference.

By Scale and Compasses.

Extend the Compasses from 1 to 4.443, that Extent will reach from 15.98 to 71, the Circumference.

Problem 16. Having the Side of a Square, to find the Circumference of a Circle that will be equal thereunto.

If the Side of a Square be 1, the Circumference of a Circle that will be equal thereunto, ſhall be 3.545. Then,

```
As 1 : 3.545 :: 20.0291
                  3.545
                -------
                1001455
               801164
             1001455
            600873
            ---------
            71.0031595 the Circumference.
```

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 3.545, that Extent will reach from 20.0291 to 71, the Circumference ſought.

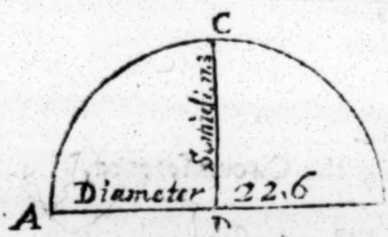
In ſeveral of the foregoing Problems, where the Diameter and Circumference are required, the Anſwers are not exactly the ſame as the Diameter and Circumference of the given Circle, but are ſometimes too much, and ſometimes too little, as in the two laſt Problems, where the Anſwers in each ſhould be 71, the one being too much, and the other too little. The Reaſon of this is, the ſmall Defect that happens to be in the Decimal Fractions, they being ſometimes too great, and ſometimes too little; yet the Defect is ſo ſmall, that it is needleſs to calculate them to more Exactneſs.

§ X. *Of a* SEMICIRCLE.

TO find the Area of the Semicircle, this is

The RULE.

Multiply the fourth Part of the Circumference of the whole Circle (that is, half the Arch Line) by the Semidiameter, the Product is the Area.



Let ABC be a Semicircle, whose Diameter 22.6, and the half Circumference, or Arch Line, A C B, is 35.5, the Half thereof is 1775, which multiply by the Semidiameter 11.3, and the Product is 200.575, the Area of the Semicircle.

```
  17.75 the half Arch Line.
   11.3 the Semidiameter.
 ------
   5325
  1775
 1775
 ------
200.575 the Area of the Semicircle.
```

By Scale and Compasses.

Extend the Compasses from 1 to 11.3; that Extent will reach from 17.75 to 200.575, the Area.

If only the Diameter of the Semicircle be given, you may say, by the *Rule of Three*,

As 1 is to .3927, so is the Square of the Diameter to the Area.

By Scale and Compasses.

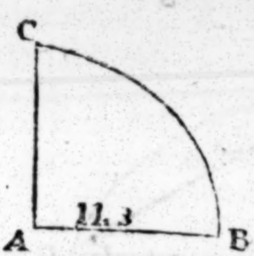
Extend the Compasses from 1 to the Diameter 22.6; that Extent turned twice from .3927, will reach, at the last, to 200.575.

§ XI. *Of a* QUADRANT.

TO find the Area of a Quadrant, or fourth Part of a Circle, this is

The RULE.

Multiply half the Arch Line of the Quadrant (that is, the eighth Part of the Circumference of the whole Circle), by the Semidiameter, and the Product is the Area of the Quadrant.



Let A B C be a Quadrant, or fourth Part of a Circle, whose Radius, or Semidiameter, is 11.3, and the half Arch Line 8.875; these multiplied together, the Product is 100.2875 for the Area.

These are the Rules and Ways commonly given for finding the Area of a Semicircle and Quadrant; but, I think, it is as good a Way, to find the Area of the whole Circle, and then take half that Area for the Semicircle, and a fourth Part for the Quadrant.

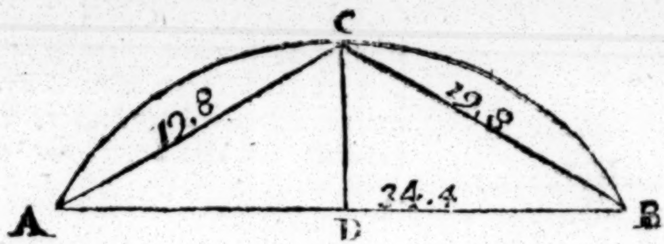
Before I proceed to shew how to find the Area of the Sector, and Segment of a Circle, I shall shew how to find the Length of the Arch Line, both *Geometrically* and *Arithmetically*.

To find the Length of the Arch Line *Geometrically.*

Divide the Chord Line AB into four equal Parts, and ſet one of theſe Parts from B to C, and draw a Line from C to three of thoſe Parts at D; ſo ſhall CD be equal to half the Arch Line ACB.

To find the Length of the Arch Line *Arithmetically.*

Multiply the Chord of half the Segment AC or CB by 8, and from the Product ſubtract the Chord of the whole Segment AB, and divide the Remainder by 3; the Quotient is the Arch Line ACB ſought.



19.8	A C
8	
158.4	
34.4	A B
3)124	
Arch Line 41.333	

Another

Another Way.

From the double Chord of half the Segment's Arch, ſubtract the Chord of the Segment, one third Part of the Difference added to the double Chord of half the Segment's Arch, the Sum is the Arch Line of the whole Segment.

Thus, if AC 19.8 be doubled, it makes 39.6; from which if you ſubtract 34.4, the Remainder is 5.2, which, divided by 3, the Quotient is 1.733; this added to 39.6 (the double Chord of the half Segment), the Sum is 41.333. So if the Arch Line ACB was ſtretched out ſtrait, it would then contain 41.333 ſuch Parts as the Chord AB contains 34.4 of the like Parts.

Theſe two Rules may very eaſily be proved out of the Table of natural Sines; thus,

Suppoſe (in the former Figure) the Arch ACB to contain 120 Degrees; the natural Sine of half, *viz.* of 60 Degrees, is 86602; which, being doubled, is 173204, which is the Chord of the whole 120 Degrees, that is, AB. Then, to find the Chord of the half Arch, *viz.* AC 60 Degrees, the half of it 30 Degrees, the natural Sine thereof is 50000; which, doubled, makes 100000 for the Chord AC; then, according to the firſt Rule, multiply 100000 by 8, the Product is 800000; from which ſubtract 173204 (the Chord AB), and the Remainder is 626796; which divide by 3, the Quotient is 208932, which is the Length of the Arch Line ACB, according to the firſt Rule. Now let us examine how near this comes to the true Quantity of the Arch propoſed. If the Radius or Semidiameter of a Circle be 100000 (as it is in the Table of Sines), then the Circumference will be 628318; and becauſe 120 Degrees in the third Part of the Circle, take the third Part of 628318 is 209439, which is the true Quantity of the Arch ACB in ſuch Parts as

the Semidiameter contains 100000, and differs from that before found 507, which is a thing inconſiderable in *Practical Menſuration.* The latter of the foregoing Rules agrees exactly with the former, and therefore the Difference will be the ſame as above; either of the Rules gives the Quantity of the Arch Line too little, and the greater the Arch, the greater the Error: If you know the Degrees that are contained in the Segment's Arch, and would have the Arch Line very exactly, you may reaſon thus by the *Rule of Three:*

As the Circle's Periphery in Degrees, is to its Periphery in equal Parts; ſo is the Arch in Degrees and decimal Parts, to the ſame Arch in equal Parts.

Suppoſe the Circumference of a Circle be 71, and ſuppoſe the Arch to contain 52 Degrees 15 Minutes (the Decimal of 15 Minutes is .25); then ſay,

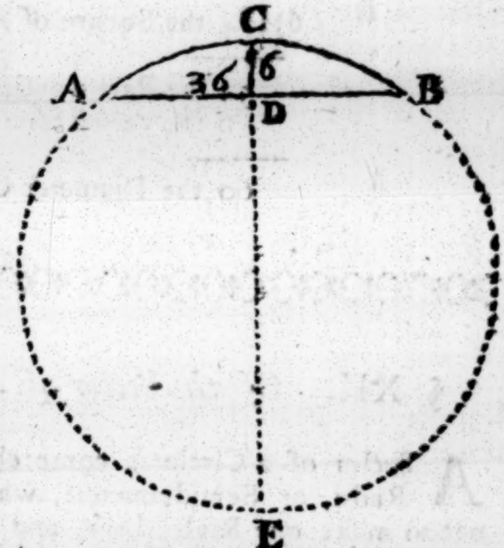
```
    Deg. Parts  Deg.
As 360 : 71 : : 52.25
                   71
                 ----
                 5225
                36575
               ------
     36|0)370|9.75(10.305 ferè.
          36
         ----
          109
          108
          ---
           175
```

So the 52 Degrees 15 Minutes will contain 10.305 of ſuch Parts as the Circumference contains 71.

Thus have I ſhewed ſeveral Ways of finding the Meaſure of the Curve Line of any Part of a Circle very near the Truth. The next thing I ſhall ſhew, is,

How

How to find the Diameter of a Circle by having the Chord and verſed Sine of the Segment Arithmetically.



Becauſe the Chord A B cuts the Diameter EC at right Angles, therefore the Semichord A D, or D B, is a mean proportional Line between the Parts of the Diameter CD and DE (by *Euc.*6.13.); therefore, if you ſquare the Semichord AD, or DB, and divide the Square by the verſed Sine CD, the Quotient will be the Part of the Diameter wanting; to which add the given verſed Sine CD, and the Sum is the Diameter ſought.

Example. Let A C B be a Segment given, whoſe Chord AB is 36, and the verſed Sine CD 6; half 36 is 18, which, ſquared, makes 324; this divided by 6, the Quotient is 54: to which add 6, the Sum is 60, the Diameter of the Circle CE.

```
    18 half the Chord.
    18
   ----
   144
    18
   ----
 6)324 the Square of AD.
   ----
    54 the Part wanting DE.
     6 the verſed Sine CD add.
   ----
    60 the Diameter CE.
```

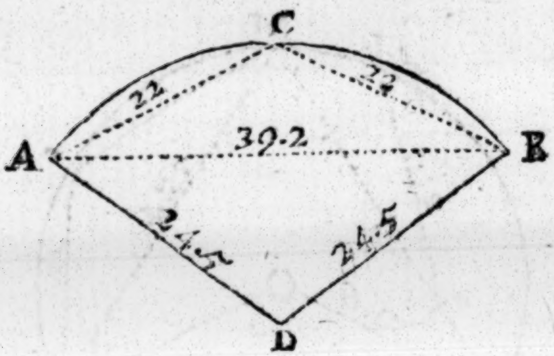
§ XII. *Of the Sector of a* Circle.

A Sector of a Circle is comprehended under two Radii, or Semidiameters, which are ſuppoſed not to make one Right Line, and a Part of the Circumference: Whence a Sector may be either leſs or greater than a Semicircle. To find the Area or ſuperficial Content thereof, this is

The RULE.

Multiply half the Arch Line by the Semidiameter, and the Product is the Area.

Let ADBC be the Sector of a Circle given, whoſe Semidiameter AD or BD is 24.5, and the Arch Line ACB (by the firſt Rule, *Pag.* 112.) I find to be 45.6; the half thereof, 22.8, being multiplied by 24.5 (the Semidiameter) the Product is 558.6; which is the Area of the Sector ACBD.

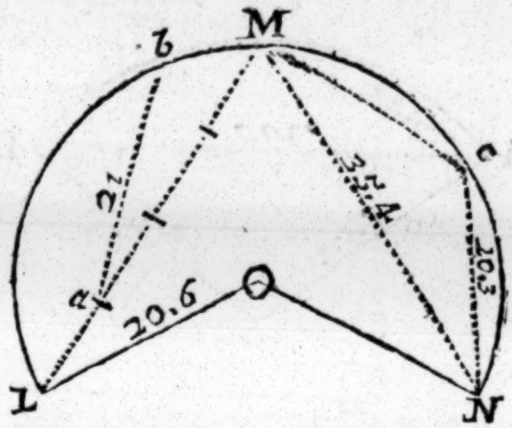


22
8
———
176
39.2 Subtrahend.
———
3)136.8
———
45.6 Arch Line.
———
22.8 half.
24.5 Semidiam.
———
1140
912
456
———
558.60 the Area.

Again: Let LMNO be a Sector greater than a Semicircle, whose Semidiameter LO or NO is 20.6. and the Line *b a* equal to a fourth Part of the Arch Line L *b* MN 21, the Double whereof is 42, equal to the Arch Line L *b* M or M *c* N; or by the arithmetical Rule, *Pag.* 112. the said Arch is found to be 42.333; which, multiplied by 20.6, the Semidiameter, makes 872.0598 for the Area of the Sector LMNO.

See the following Work.

20.3



20.3	Chord N c.	42.333	
8		20.6	
162.4		253998	
35.4	Chord MN.	84666	
3)12.70		872.0598	Area.
42.33	Arch Line.		

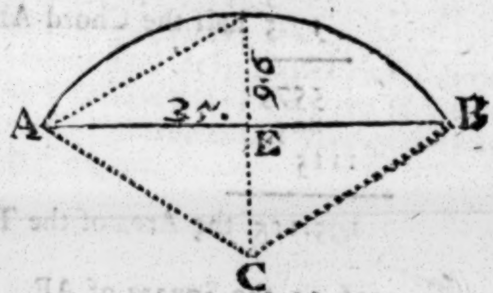
§ XIII. *Of the Segment of a* Circle.

A Segment of a Circle is a Part terminated by a Right Line lefs than the Diameter, called a Chord, and by a Part of the Circumference.

To find the Area of the Segment of a Circle, you muft, firft, find the Center of the whole Circle, and draw the two Semidiameters, thereby completing the Sector, as in the following Figure. Then (by the laft Section,) find the Area of the whole Sector CADBC, and then (by Sect. 5.) find the Area of the Triangle ABC, and fubtract the Area of the Triangle out of the Area of the Sector, the Remainder is the Area of the Segment.

Other-

Otherwiſe you may, without deſcribing the Figure, find the Semidiameter of the Circle by the arithmetical Rule (*pag.* 113.) and by the arithmetical Rule (*pag.* 112.) find the Arch Line; then multiply half the Arch Line by the Semidiameter; ſo have you the Area of the Sector: Then ſubtract the verſed Sine from the Semidiameter, the Remainder is the Perpendicular of the Triangle; and multiply the half Chord by the Perpendicular, the Product is the Area of the Triangle: Then ſubtract the Area of the Triangle from the Area of the Sector, and the Remainder is the Area of the Segment. See the Work.



2)35=AB

17.5
17.5

875
1225
175

9.6)306.25(31.9
9.6 add.

182
865 41,5 the Diameter of the Circle.

1 20.75 the Semidiameter.
9.6 DE Subtrahend.

11.15 remains the Perpendicular EC.

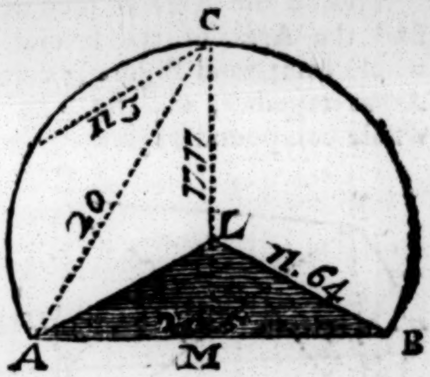
```
   11.15 the Perpendicular EC.
    17.5 half the Chord AE or EB.
   -----
    5575
   7805
  1115
  -------
 195.125 the Area of the Triangle.

  306.25 the Square of AE.
   92.16 the Square of DE the verfed Sine.
  ------
  398.41 Sum.

The Square Root thereof is 19.96 the Chord AD
                                8
                           ------
                           159.68
                      Sub.  35     the Chord AB.
                          -------
                         3)124.6
                          -------
                         2)41.56 the Arch Line.
                          -------
                           20.78 half.
                           20.75 Semidiameter.
                          -------
                           10390
                          14546
                         4156
                         --------
From                     431.1850 Area of the Sect.
Subtract                 195.125  Area of the Tri.
                         --------
Remains                  236.060  Area of the Seg.
```

Again:

Again: Let MACBM be a Segment greater than a Semicircle; obſerve the former Rules, in all reſpects, as in the laſt Example; only, inſtead of ſubtracting the Area of the Triangle out of the Area of the Sector, here you muſt add it thereunto, as may plainly appear by the following Figure.



11.5
8
———
92.0
20
———
3)72
———
24 Half Arch line.

11.64 Semidiam.
24
———
4656
2328
———
279.36 Area of the Sector LACBL.

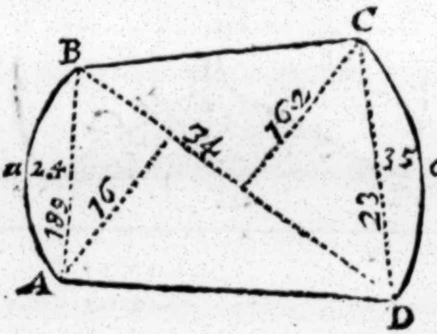
17.17
11.64
———
5.53 LM.

10.25 half the Baſe MA.
5.53 the Perpendicular LM.
———
3075
5125
5125
———
56.6825 the Area of the Triangle ALM;
279.36 the Area of the Sector add.
———
336.0425 the Area of the Segment ſought.

§ XIV. *Of Compound* Figures.

MIXED or compound Figures are ſuch as are compoſed of rectilineal and curvilineal Figures together.

To find the Area of ſuch mixed Figures, you muſt find the Area of the ſeveral Figures of which the whole compound Figure is compoſed, and add all the Areas together, and the Sum will be the Area of the whole compound Figure.



16.2 } add.
16

32.2 Sum.

16.1 half.
34 Diagonal.

644
483

547.4 Area of the (Trapez

10.236 half the Arch Line A a B.
14.83 Semidiameter of the Arch AB.

30708
81888
40944
10236

151.79988 Area of the Sector.

From

From 14.83 Semidiameter
Subtract 3.4 Verſed Sine.

11.43 Perpend. of the Triangle.
9.45 half the Chord AB.

5715
4572
10287

108.0135 the Area of the Tria. ſubtracted from
151 7999 the Area of the Sector.

43.7864 the Area of the Segment A a B A.

12.19 half the Arch Line C c D.
20.64 Semidiameter.

4876
7314
24380

251.6016 the Area of the Sector.

From 20.64 the Semidiameter
Subtract 3 5 Verſed Line.

Remain. 17.14 Perpendicular of the Triangle.
11 5 half the Chord DC.

8570
1714
1714

Sub. 197.110 Area of the Triangle
From 251.602 Area of the Sector.

Rem. 54.492 the Area of the Segment Cc DC.
43.786 the Area of the Segment Aa BA.
547.4 the Area of the Trapezium.

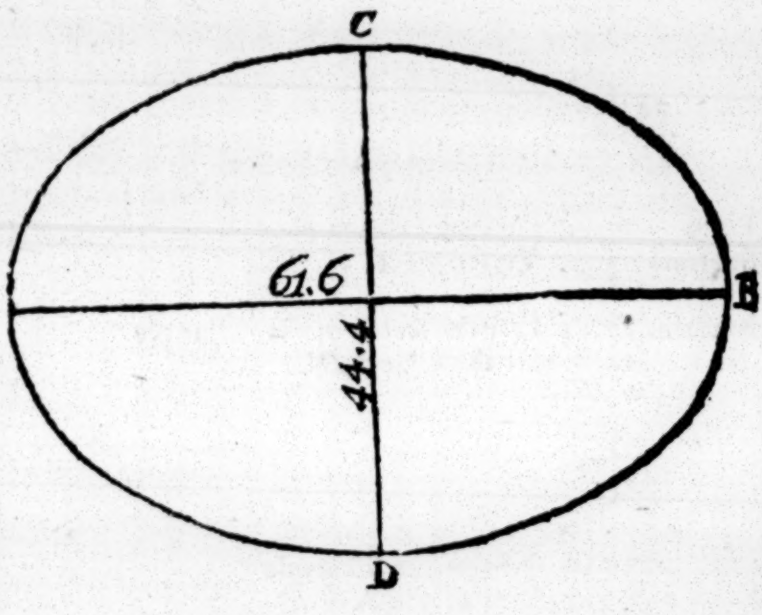
Sum 645.678 the Area of the Whole.

§ XV. *Of an* Ellipsis.

AN Ellipfis, or Oval, is a Figure bounded by a regular Curve Line, returning into itfelf; but of its two Diameters, cutting each other in the Center, one is longer than the other, in which it differs from the Circle. To find the Area thereof, this is

The RULE.

Multiply the traverfe Diameter by the Conjugate, and multiply that Product by .7854, this laft Product is the Area of the Oval.



61.6

```
      61.6 the traverse Diameter.
      44.4 the conjugate Diameter.
     ------
      2464
     2464
    2464
    ------
   2735.04 the Rectangle.
     .7854 the Area of Unity.
   --------
   1094016
  1367520
 2188032
1914528
-----------
2148.100416 the Area of the Oval.
```

Demonstration. If you circumscribe any Ellipsis with a Circle, and suppose an infinite Number of Chord Lines drawn therein, all parallel to the conjugate Diameter, as these in the following Figure; then it will be,

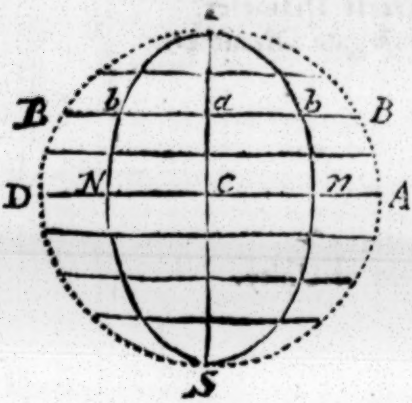
As DA, the Diameter of the Circle, is to N n, the conjugate Diameter of the Ellipsis; so is B a B, any Chord in the Circle, to b a b, its respective Ordinate in the Ellipsis.

For, according to the Property of the Circle,

it is	1	$a S \times T a = \square B a$.
And		by the Property of the Ellipsis,
it is	2	$\square TC : \square NC :: a S \times T a : \square b a$.
1, 2,	3	$\square TC : \square NC :: \square B a : \square b a$.
3, hence	4	$TC : NC :: B a : b a$.
Conseq.	5	$TC : 2 NC :: 2 B a : 2 b a$.
That is	6	$DA : Nn :: B a b : b a b$.

But the Sum of an infinite Series of such Chords as B a b, do constitute the Area of the Circle. And the Sum of the like Series of their respective Ordinates, as b a b, do constitute the Area of the Ellipsis:

 There-



Therefore TS : Nn : : Circle's Area : the Ellipſis Area. But TS : Nn : : □ TS : TS × Nn ; whence it follows, that,

□ TS : Circle's Area : : TS × Nn : Ellipſis Area.

Conſequently, As 1 is to .7854, ſo is the Rectangle, or Product, of the tranſverſe and conjugate Diameter of any Ellipſis to its Area.

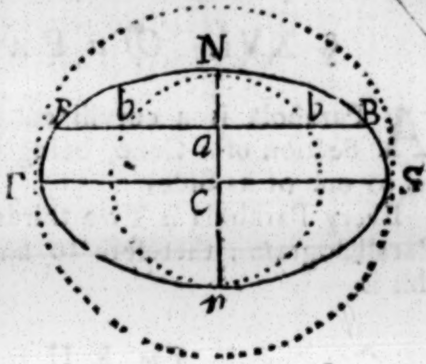
Hence it is eaſy to conceive, that the Square Root of the Product of the tranſverſe and conjugate Diameters will be the Diameter of a Circle equal to the Ellipſis.

Hence alſo the Segments of an Ellipſis, and its circumſcribing Circle (whoſe Baſes are parallel to the conjugate Diameter, and of the ſame Height), are in Proportion one to another as their Baſes are. That is,

B a B : b a b : : Area Segment BTB : Area Segment b TB.

Or, TS : N n : : Area Segment BTB : Area Segment b T b.

The Area of every Ellipſis is a mean Proportional between the Area of its circumſcribing and inſcribed Circles.



The Truth of this may easily be deduced from the last; for 'tis already proved, that $\square$ TS : TS $\times$ Nn :: circumscribing Circle's Area : Ellipsis Area.

But $\square$ TS : TS $\times$ Nn : : TS $\times$ Nn : $\square$ Nn. Therefore Ellipsis Area : inscribed Circle's Area,: TS $\times$ Nn: $\square$ Nn.

Example. Let TS$=$36, and Nn$=$18.4.
Then $\square$ TS$=$1296, and $\square$ Nn$=$338.56.
Then 1296$\times$.7854$=$1017.8784 great Circle's Area;
And 338.56$\times$.7854$=$265.905, &c. lesser Circle's Area;
And 36$\times$18.4$=$662 4$\times$.7854$=$520.24896, which is the Area of the Ellipsis; then it will be,

1017.878 : 520.24896 : : 520.24896 : 265.905024.

That is, As the great Circle's Area is to the Area of the Ellipsis, so is the Area of the Ellipsis to the Area of the lesser Circle.

From hence it follows, that all Segments of an Ellipsis, and its inscribed Circle (whose Bases are parallel to the transverse Diameter, and have the same Height) are in Proportion one to another as the Area of the Ellipsis and Circle are.

That is, as the Area of the Circle is to the Area of the Ellipsis, so is the Segment b N b : to the Segment BNB;

Or, Nn : TS : : Area Segment b N b : Area Segment BNB.

§ XVI. Of

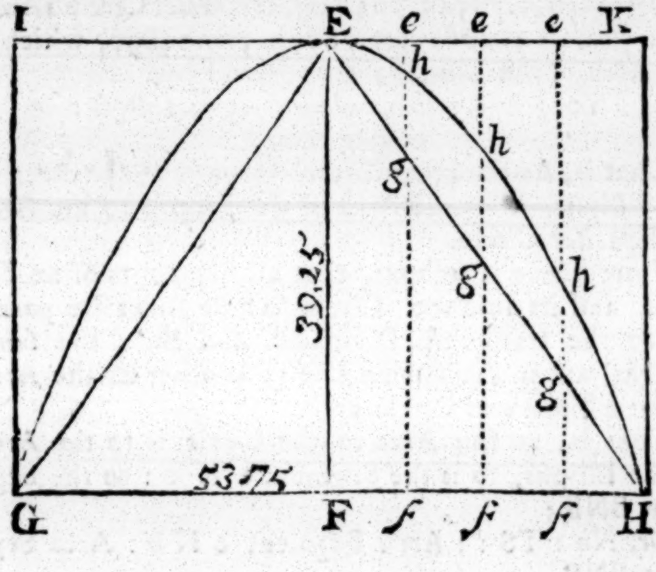
§ XVI. *Of a* PARABOLA.

A Parabola is a curvilineal Figure, made by the Section of a Cone, being cut by a Plane parallel to one of its Sides.

Every Parabola is Two-thirds of its circumſcribing Parallelogram; therefore to find the Area thereof, this is

The RULE.

Multiply the Baſe, or greateſt Ordinate, by the perpendicular Height, and multiply that Product by 2, and divide the laſt Product by 3, the Quotient will be the Area of the Parabola.



$$\begin{array}{rl}
53.75 & \text{the Ordinate GH.} \\
39.25 & \text{the Perpendicular EF.} \\
\hline
26875 & \\
10750 & \\
48375 & \\
16125 & \\
\hline
2109.6875 & \\
2 & \\
\hline
3)4219.3750 & \\
\hline
1406.4583 & \text{the Area.}
\end{array}$$

Demonstration. Let FH, the Semi-ordinate, be divided into four equal Parts, or into 8.16, &c. and through the Divisions draw Lines, as e f, e f, &c. parallel to the Axis EF. Suppose also EF to be 4.

Then, I say, the Parabolic Sphere Eh HF is to the Parallelogram EKFH as 2 to 3; but to the Triangle EFH as 4 to 3.

For, first, g f, g f, g f, &c. are in continual arithmetical Proportion from the Nature of plain Triangles.

Secondly, f e : g e : : g e : h e; but h e is the Axis EF=0; and in the first Parallel e f must be equal to $\frac{1}{4}$, in the next e f must be equal to $\frac{4}{4}$, in the third to $\frac{9}{4}$, and so on, in a duplicate arithmetical Progression.

For e f (=4) : g e (=1) : : g e (=1) : e h (=$\frac{1}{4}$). And the second e f (=4) : e g (=2) : : e g (=2) : e h (=$\frac{4}{4}$), &c. And thus it will be, if the Lines F f, f f, &c. be again bisected, &c. *ad infinitum*, so that all the Indivisibles of the trilinear Space EK H h E will be in a duplicate arithmetical Progression increasing. But the Sum of a Rank of such Terms is subtriple to a Rank of as many equal to the greatest (by Lemma 3); wherefore the whole trilinear Space E K H h E is to the Parallelogram as 1 to 3; and, consequently, the remaining parabolic Space must be to it as 2 to 3; which was to be proved.

And

And ſince the Triangle FEH is to the Parallelogram as 1 to 2, it muſt be to the Parabola as $1\frac{1}{2}$ to 2, or as 3 to 4; which was to be proved.

Before I proceed to the Menſuration of ſolid Bodies, I will lay down ſuch Lemmas as will be neceſſary to facilitate the Demonſtration of all ſuch Solids.

LEMMA I.

In any Series of equal Numbers (repreſenting Lines or other Quantities), as 1, 1, 1, 1, *&c.* or 2, 2, 2, 2, *&c.* or 3, 3, 3, 3, *&c.* if one of the Terms be multiplied into the Number of Terms, the Product will be the Sum of all the Terms in the Series.

LEMMA II.

If a Series of Numbers, in arithmetical Progreſſions, begin with a Cypher, and the common Difference be 1, as 0, 1, 2, 3, *&c.* (repreſenting a Series of Lines or Roots beginning with a Point) if the laſt Term be multiplied into the Number of Terms, the Product will be double the Sum of all the Series.

That is, putting L=the laſt Term, N=the Number of Terms, and S=the Sum of all the Series; then will NL=2 S; conſequently, $\frac{1}{2}$ NL=S; *viz.* One half of ſo many times the greateſt Term as there are Numbers of Terms in the Series.

$$\text{Thus}\begin{cases} 0+1+2+3+4=10 \text{ the Sum}=\frac{1}{2}\text{NL.} \\ 4+4+4+4+4=20=\text{NL.}\end{cases}$$

LEMMA III.

If a Series of Squares, whoſe Sides or Roots are in arithmetical Progreſſion, beginning with a Cypher *&c.* be infinitely continued, the laſt Term, being multiplied into the Number of Terms, will be triple

to

to the Sum of all the Series; *viz.* $NLL = 3S$; or, $\frac{1}{3}$ $NLL = S$.

That is, the Sum of ſuch a Series will be One-third of the laſt or greateſt Term, ſo many times repeated as there are Numbers of Terms in their Series.

Inſtances in ſquare Numbers.

$$1 \left\{ \frac{0+1+4}{4+4+4} = \frac{5}{12} = \frac{1}{3} + \frac{1}{12}. \right.$$

$$2 \left\{ \frac{0+1+4+9}{9+9+9+9} = \frac{14}{56} = \frac{7}{18} = \frac{1}{5} + \frac{1}{18}. \right.$$

$$3 \left\{ \frac{0+1+4+9+16}{10+16+10+10+16} = \frac{30}{80} = \frac{3}{8} = \frac{9}{24} = \frac{1}{3} + \frac{1}{15} \right. \text{(\&c.}$$

From theſe Inſtances it is evident, that as the Number of Terms in the Series do increaſe, the Fraction or Exceſs above $\frac{1}{3}$ does increaſe, the ſaid Exceſs always being $\frac{1}{6N-6}$; which, if we ſuppoſe the Series to be infinitely continued, will quite vaniſh, and become nothing at all.

LEMMA IV.

If a Series of Cubes, whoſe Roots are in arithmetical Progreſſion, beginning with a Cypher, *&c.* (as above) be infinitely continued, the Sum of all the Series will be $\frac{1}{4}$ $NLLL = S$.

That is, One fourth of the laſt Term ſo many times repeated as there are Numbers of Terms.

Inftances in cube Numbers.

If 0, 1, 2, 3, 4, 5, *&c.* be the Roots of the Cubes,

$$1 \left\{ \frac{0+1+8+27}{27+27+27+27} = \frac{36}{108} = \frac{4}{12} = \frac{1}{4} + \frac{1}{12}. \right.$$

$$2 \left\{ \frac{0+1+8+27+64}{64+64+64+64+64} = \frac{100}{320} = \frac{10}{32} = \frac{5}{10} = \frac{1}{4} + \frac{1}{16}. \right.$$

$$3 \left\{ \frac{0+1+8+27+64+125}{125+125+125+125+125+125} = \frac{125}{750} = \frac{45}{156} \right.$$

$$(= \frac{1}{20} = \frac{1}{4} = \frac{1}{20}.$$

From thefe Examples it plainly appears, that as the Number of Terms in the Series increafes, the Fraction or Excefs above $\frac{1}{4}$ decreafes, the Excefs being always $\frac{1}{4N-4}$; which, if we fuppofe the Series to be infinitely continued, will become infinitely fmall, or nothing.

LEMMA V.

If a Series of Biquadrates, whofe Roots are in arithmetical Progreffion, beginning with a Cypher, *&c.* as before, be infinitely continued, the Sum of all the Terms in fuch a Series will be $\frac{1}{5}$ NLLLL.

The Truth of this may be manifefted by the like Procefs as in the foregoing Lemmas, and fo on for higher Powers.

LEMMA VI.

The Sum of an infinite Progreffion, whofe greateft Term is a fquare Number, the other decreafing by odd Numbers; *viz.* 1, 3, 5, *&c.* is in fubfefquialteran Proportion of the Sum of the like Number of equal Terms, that is, as 2 to 3.

Inftances

Inſtances in ſuch Progreſſions.

$$1 \left\{ \frac{9+8+5}{9+9+9} = \frac{22}{27} = \frac{2}{3} + \frac{4}{9} \right.$$

$$2 \left\{ \frac{16+15+12+7}{16+16+16+16} = \frac{50}{64} = \frac{2}{3} + \frac{11}{32}. \right.$$

$$3 \left\{ \frac{25+24+21+16+9}{25+25+25+25+25} = \frac{95}{125} = \frac{2}{3} = \frac{7}{27}. \right.$$

$$4 \left\{ \frac{36+35+32+27+20=11}{36+36+36+36+36+36} = \frac{161}{206} = \frac{2}{3} + \frac{17}{72} \right.$$

From theſe Examples it plainly appears, that, as the Number of Terms in the Series increaſes, the Fraction or Exceſs above $\frac{2}{3}$ decreaſes; and if we ſuppoſe the Series to be infinitely continued, that Exceſs will quite vaniſh, and the Sum of the infinite Series will be $\frac{2}{3}$ of ſo many equal to the greateſt.

CHAP. II.

The Menſuration of SOLIDS.

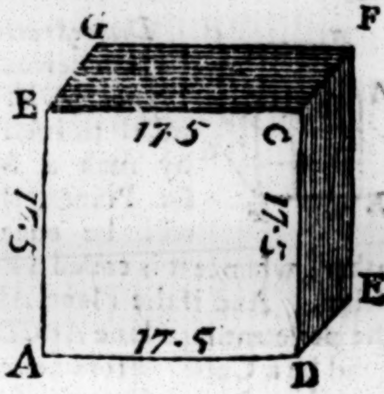
SOLID Bodies are ſuch as do conſiſt of Length, Breadth, and Thickneſs; as Stone, Timber, Globes, Bullets, *&c.*

§ I. *Of a* CUBE.

A CUBE is a ſquare Solid, comprehended under ſix geometrical Squares, being in the Form of a Dye. To find the ſolid Content, this is

The RULE.

Multiply the Side of the Cube into itſelf, and that Product again by the Side; the laſt Product will be the Solidity, or ſolid Content of the Cube.



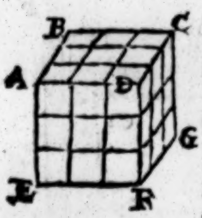

```
   17·5
   17·5
  ------
    875
  1225
  175
  ------
  306 25
    17·5
  ------
  153125
 214375
 30625
 --------
5359·375
```

5359·375 the ſolid Content of the Cube.

Suppoſe ABCDEFG a cubical Piece of Stone or Wood, each Side thereof being 17 Inches and an half; multiply 17.5 by 17.5, and the Product is 306.25; which being multiplied by 17.5, the laſt Product is 5359·375, which is 5359 ſolid Inches and 375 Parts. To reduce the ſolid Inches to Feet, divide by 1728 (becauſe ſo many cubical Inches is a Foot), and the ſolid Feet in the Cube will be 3, and 175 cubical Inches remain.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 17.5; that Extent turned over twice from 17.5 will reach to 5359, the ſolid Content in Inches. Then extend the Compaſſes from 1728 to 1; that Extent, turned the ſame Way from 5359, will reach to 3.1 Feet.



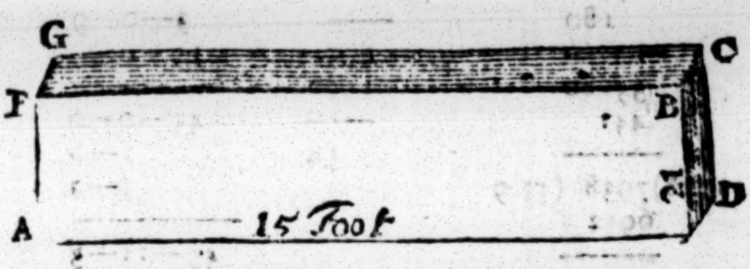
Demonſtration. If the Square ABCD be conceived to be moved down the Plane ADEF, always remaining parallel to itſelf, there will be generated, by ſuch a Motion, a Solid, having ſix Planes, the two oppoſite whereof will be equal and parallel to each other; whence it is called a Parallelopipedon, or ſquare Priſm. And if the Plane ADEF be a Square equal to the penetrating Plane ABCD, then will the generated Solid be a Cube. From hence ſuch Solids may be conceived to be conſtituted of an infinite Series of equal Squares, each equal to the Square ABCD; and AE or DF will be the Number of Terms Therefore, if the Area of ABCD be multiplied into the Number of Terms AE, the Product is the Sum of all the Series, (*per* Lemma I.) and, conſequently, the Solidity of the Parallelopipedon or Cube. Or, if the Baſe ABCD, being divided into little ſquare Areas, be multiplied into the Height AE, divided by a like Meaſure for Length, after this Way you may conceive as many little Cubes to be generated in the whole Solid, as is the Number of the little Areas of the Baſe multiplied by the Number of Diviſions the Side AE contains. Thus, if the Side of the Baſe AB be 3, that multiplied into itſelf is 9, which is the Area of the ſquare Baſe ABCD; then, if AE be likewiſe 3, multiply 9 by 3, and the Product is 27; and ſo many little Cubes will this Solid be cut into, if you conceive it to be cut as the Lines direct.

From this Demonſtration it is very plain, that, if you multiply the Area of the Baſe of any Parallelopipedon into its Length or Height, that Product will be the ſolid Content of ſuch a Solid.

§ II.

§ II. *Of a* PARALLELOPIPEDON.

LET ABCDEFG be a Parallelopipedon, or ſquare Priſm, repreſenting a ſquare Piece of Timber or Stone, each Side of its ſquare Baſe ABCD being 21 Inches, and its Length AE 15 Feet.



Firſt, then, multiply 21 by 21, the Product is 441, the Area of the Baſe in Inches; which multiplied by 180, the Length in Inches, and the Product is 79380, the ſolid Content in Inches. Divide the laſt Product by 1728, and the Quotient is 45.9, that is, 45 ſolid Feet and 9 Tenths of a Foot. Or thus: Multiply 441 by 15 Feet, and the Product is 6615; divide this by 144, and the Quotient is 45.9, the ſame as before.

Or thus, by multiplying Feet and Inches.

Multiply 1 Foot 9 Inches by 1 Foot 9 Inches, and the Product is 3 Feet 0 Inches 9 Parts; this multiplied again by 15 Feet, gives 45 Feet 11 Inches 3 Parts, that is, 45 Feet and $\frac{11}{12}$ of a Foot and $\frac{1}{4}$ of $\frac{1}{12}$.

See the Work of all theſe.

21	441	F. I.
21	15	1—9
—	—	1—9
21	2205	—
42	441	1—9
—	—	1—3—9
441	144)6615(45 9	—
180	—	3—0—9
—	855	15
35280	1350	—
441	—	45—0—0
—	54	7—6
1728)79380(45·9		3—9
6912		—
—		45—11—3
10260		
8640		
—		
16200		
15552		
—		
648		

By Scale and Compaſſes.

Extend the Compaſſes from 12 to 21, and that Extent will reach to near 46 Feet, being twice turned over from 15 Feet; ſo the ſolid Content is almoſt 46 Feet.

If the Baſe of the ſquared Solid be not an exact Square, but in Form of a rectangle Parallelogram, the Way of meaſuring it is much the ſame; for, firſt, you muſt find the Area of the Baſe by multiplying the Breadth by the Depth; and then multiply that Area by the Length of the Piece, as before. Thus,

If a Piece of Timber be 25 Inches broad, 9 Inches deep, and 25 Feet long, how many folid Feet are contained therein ?

25	F. I.
9	2—1
	0—9
225	
25	1—6—9
	25
1125	
450	25—0—0
	12—6—0
144)5625(39	1—0—6
432	0—6—3
1305	39—0—9
1296	
9	Anfwer 39 Feet.

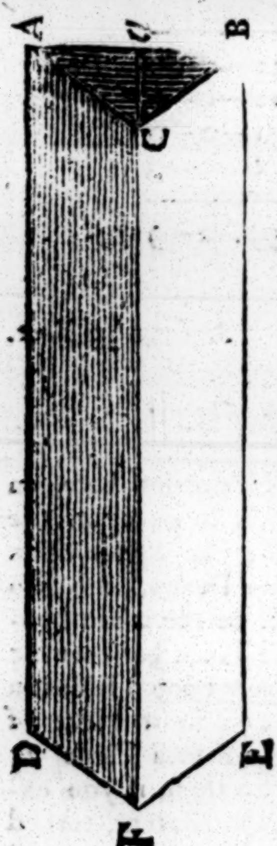
By Scale and Compaffes.

Firft, find a mean geometrical Proportion between the Breadth and the Depth ; which to do upon the Line of Numbers, you muft divide the Space upon the Line, between the Breadth and Depth, into two equal Parts ; that middle Point will be the mean Proportional fought : Thus the middle Point between 25 and 9 is at 15 ; fo is 15 a mean Proportional between 9 and 25, for, 9 : 15 : : 15 : 25 ; fo a Piece of Timber of 15 Inches fquare is equal to a Piece 25 Inches broad and 9 Inches deep. So then, if you extend the Compaffes from 12 to 15, that Extent, turned twice over from 25 Feet, the Length, will reach to 39 Feet, the Content.

§ III.

§ III. *Of a Triangular* PRISM.

A Prifm is a Solid contained under feveral Planes, and having its Bafes like, equal, and parallel. The folid Content of a Prifm (whether triangular or multangular) is found by multiplying the Area of the Bafe into the Length or Height, and the Product is the folid Content.



Let ABCDEF be a triangular Prifm, each Side of the Bafe being 15.6 Inches, the Perpendicular thereof C *a* is 13.51 Inches, and the Length of the Solid 19.5 Feet.

Multiply the Perpendicular of the Triangle 13.51 by half the Side 7.8, and the Product is 105.378, the Area of the Bafe; which multiply by the Length 19.5, and the Product is 2054.871; which divide by 144, and the Quotient is 14.27 Feet *fere*, the folid Content.

13.51	144)2054.87(14.27
7.8	144
10808	614
9457	576
105.378	388
19.5	288
526890	1007
948402	1008
105378	
2054.8710	

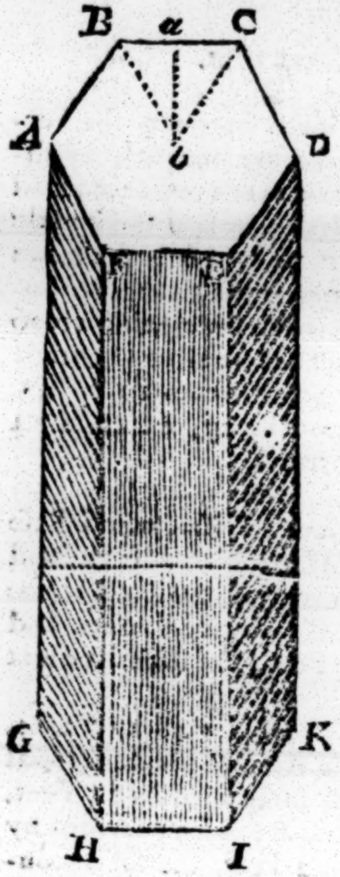
By

By Scale and Compaſſes.

Firſt, find a mean Proportional between the Perpendicular and Half-ſide (as before taught), by dividing the Space upon the Line, between 13.51 and 7.8 into two equal Parts; ſo ſhall you find the middle Point between them to be at 10.26, which is the mean Proportional ſought: By this means the triangular Solid is brought to a ſquare one, each Side being 10.26 Inches. Then extend the Compaſſes from 12 to 10.26; that Extent, turned twice downwards from 19.5 Feet, the Length will at laſt fall upon 14.27, which is 14 Feet and a little above a Quarter.

Let ABCDEFGHIK repreſent a Priſm, whoſe Baſe is a Hexagon, each Side thereof being 16 Inches, and the Perpendicular from the Center of the Baſe to the Middle of one of the Sides (a b) is 13.14 Inches, and the Length of the Priſm is 15 Feet; the ſolid Content is required.

Multiply half ihe Sum of the Sides 48 by 13.84, and the Product is 664.32, the Area of the hexagonal Baſe (by § VIII. *p.* 86), which multiplied by 15 Feet, the Length, the Product is 9964.8; which divided by 144, the Quotient will be 69.2 Feet, the ſolid Content required.



13.84
48

11072
5536

664.32 Ar. of Bafe.
15

332160
66432

144)9964.80(69.2
864

1324
1296

288
228

By Scale and Compaffes.

Firft, find a mean Proportional between the Perpendicular, and half the Sum of the Sides; that is, divide the Space between 13.84 and 48, and the middle Point will be 25.77. Then extend the Compaffes from 12 to 25.77; that Extent will reach (being twice turned over) from 15 Feet, the Length, to 69.2 Feet, the Content.

To find the ſuperficial Content of any of the fore-mentioned Solids, you muſt take the Girth of the Piece, and multiply by the Length, and to that Product add the two Areas of the Baſes, the Sum will be the whole ſuperficial Content. Example of the hexagonal Priſm laſt-mentioned. The Sum of the Sides being 96, and the Length 15 Feet, that is, 180 Inches; which multiplied by 96, the Product is 17280 ſquare Inches; to which add twice 664.32, the Areas of the two Baſes, the Sum is 18608.64, the Area of the Whole, which is 129.22 Feet.

```
        180
         96
      -----
       1080
      1620
      -----
      17280
        664.32
        664.32
     --------
144)18608.64(129.22
     -----
      420
      1328
        326
         384
        ----
          96
```

The ſuperficial Content of the whole Solid is 129.22 Feet.

By Scale and Compaſſes.

Extend the Compaſſes from 144 to 180; that Extent will reach from 96 to 120 Feet. Then, to find the Area of the Baſe, extend the Compaſſes from 144 to

to 13.84; that Extent will reach from 48 to 4.6 Feet; add 120 Feet, and twice 4.6 Feet, and it makes 129.2 Feet, the fuperficial Content, as before.

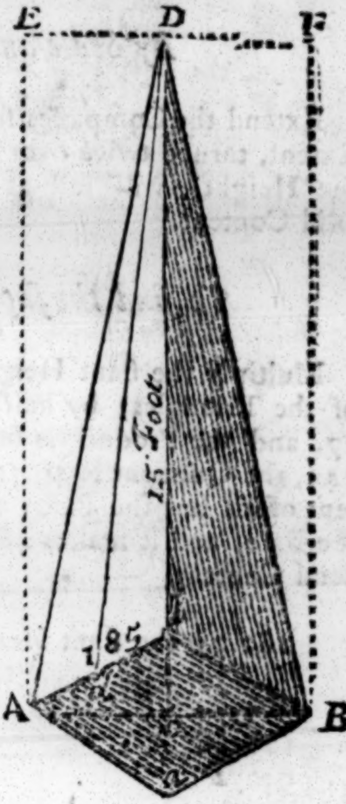
The Demonftration of thofe laft Solids will be the fame as in the firft Section; for as in that, fo in thefe, the Area of the Bafe is multiplied into the Length to find the Content, and the fame Reafon is given for one as for the other.

§ IV. *Of a* Pyramid.

A Pyramid is a folid Figure, whofe Bafe is a Polygon, and whofe Sides are plain Triangles, their feveral Tops meeting together in one Point. To find the folid Content thereof, this is

The RULE.

Multiply the Area of the Bafe by a third Part of the Altitude, or Length; and the Product is the folid Content of the Pyramid.



Let A B D be a ſquare Pyramid, each Side of the Baſe being 18.5 Inches, and the perpendicular Height CD is 15 Feet: Multiply 18.5 by 18.5, and the Product is 342.25, the Area of the Baſe in Inches; which, multiplied by 5, a third Part of the Height, and the Product is 1711.25; this divided by 144, the Quotient is 11.88 Feet, the ſolid Content.

```
   18.5
   18.5
  ------
    925
  1480
  185
  ------
  342.25 Area of the Baſe.
       5
  ------
144)1711.25(11.88 Content.
```

F.	I.	Pts.	
1	6	6	
1	6	6	
1	6	6	
	9	3	
		9	3
2	4	6	3
			5
11	10	7	3

By Scale and Compaſſes.

Extend the Compaſſes from 12 to 18.5 Inches, that Extent, turned twice over from 5 Feet (a third Part of the Height), will fall at laſt upon 11 88 Feet, the ſolid Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe 37, and the Product is 6668.88; which divided by 144, the Quotient is 46.31 Feet, the ſuperficial Content of all but the Baſe; then to that add 2.38 Feet, the Baſe, and it makes 48.69 Feet, the whole ſuperficial Content.

180.24 the ſlant Height d D.
37

126168
54072

844)6668.88(46.31
576

908
864

448
432

168
144

24

2.38

4896 the whole Content.

144)342.25(2.38
288

542
432

1105
1152

By

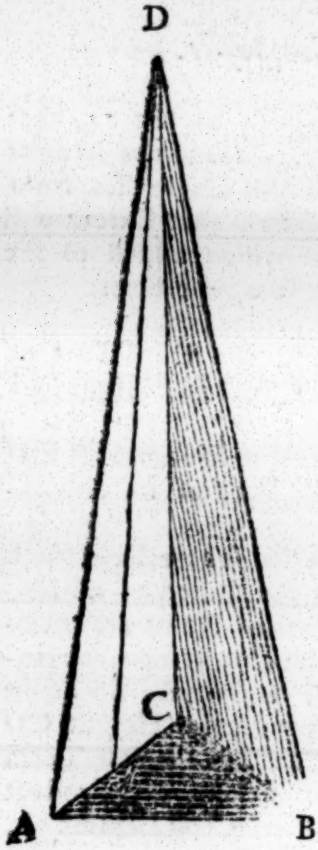
By Scale and Compaſſes.

Extend the Compaſſes from 144 to 180.24, that Extent will reach from 37 to 46.31 Feet, the Area of the four Triangles; and extend the Compaſſes from 144 to 18.5; (one Side of the Baſe), that Extent will reach from 18.5 to 2.38 *ferè:* which added to the other, the Sum is 48.69, the whole Superficies.

Demonſtration. Every Pyramid is a third Part of the Priſm, that hath the ſame Baſe and Height (by *Eucl.* 12.7.)

That is, the ſolid Content of the Pyramid A B D (in the laſt Figure) is one third Part of its circumſcribing Priſm ABEF.

For every Pyramid that hath a ſquare Baſe (ſuch as Aa Bb in the laſt Figure) is conſtituted of an infinite Series of Squares, whoſe Sides or Roots are continually increaſing in arithmetical Progreſſion, beginning at the Vertex or Point D, its Baſe Aa Bb being the greateſt Term, and its perpendicular Height CD is the Number of all the Terms: But the laſt Term multiplied into the Number of Terms, the Product will be triple the Sum of all the Series (by Lemma 3); conſequently $\frac{NLL}{3} = S$. And S is equal to the ſolid Content of the Pyramid. From hence it will be eaſy to conceive, that every Pyramid is $\frac{1}{3}$ of its circumſcribing Priſm (that is, of a Priſm of equal Baſe and Altitude), what Form ſoever its Baſe is of; *viz.* whether it be ſquare, triangular, pentangular, *&c.* You may very eaſily prove a triangular Pyramid to be a third Part of a Priſm of equal Baſe and Altitude, by cutting a triangular Priſm of Cork, and then cut that Priſm into three Pyramids, by cutting diagonally, as I have ſeveral times done, to ſatisfy myſelf and others.



Let ABCD be a triangular Pyramid, each Side of the Baſe being 21.5 Inches, and its perpendicular Height 16 Feet; the Content, ſolid and ſuperficial, is required.

Firſt, find the Area of the Baſe, by multiplying half the Side of the Perpendicular let fall from the Angle of the Baſe to the oppoſite Side; which Perpendicular will be found to be 18.62; the Half thereof is 9.31, multiplied by 21.5, the Product is 200.165 Inches, the Area of the Baſe. Then, becauſe the Altitude 16 cannot exactly be divided by 3, therefore I take the third Part of 200.165, which is 66.72, and multlply it by 16, and the Product is 1067.52; which divided by 144, the Quotient is 7.41 Feet, the ſolid Content.

9.31 half the Perpend.		F.	I.	Pts.	
21.5 the Side.	Side	1	6	6	
———	Half Perp.		9	4	
4655					
931		1	4	1	6
1862				7	2
———					
3)200.165 Area Baſe.	Area Baſe	1	4	8	8
———					4
66.72 a third Part.					
16 Height.		5	6	10	8
———					4
40032					
6672		3)22	3	6	8
———					
144)1067.52(7.41 Solid Cont.	Cont.	7	5	2	2
1008					
———					
595					
576					
———					
192					
144					
———					
48					

In caſting this up by Feet and Inches, inſtead of multiplying by 16, the Height, I break 16 into two ſuch Numbers, as, being multiplied together, the Product may be 16; *viz.* into 4 and 4, and multiply firſt by one, and then the other; a third Part of the laſt Product is the Content.

By Scale and Compaſſes.

Firſt, find a geometrical mean Proportional (as before directed), by dividing the Space between 21.5 and 9.31 into two equal Parts, and you will find the middle Point at 14.15, which is the mean Proportional ſought. Then extend the Compaſſes from 12 to

14.15, that Extent (turned twice over from 16 Feet) will fall at laſt upon 22.23; a third Part thereof is 7.41 Feet, the Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe, and to that Product add the Area of the Baſe, the Sum is the whole ſuperficial Content.

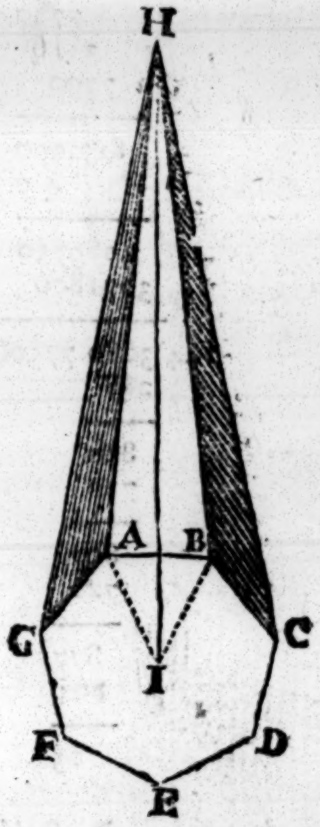
```
                192.1 Inches, the ſlant Height d D.
 Half Periph.  32.25=21.5+1075
              -------
                 9605
                3842
               3842
              5763
              --------
              6195.225 Inches, the Area of all but the
               200.165 Area of the Baſe add.     (Baſe.
              --------
          144)6395.390(44.41 Feet, the whole Content.
              576
              ---
               635
              576
              ----
               593
               576
               ----
                179
                144
                ---
                 35
```

By Scales and Compaſſes.

Extend the Compaſſes from 144 to 192.1, that Extent will reach from 32.25 (half the Periphery of the Baſe) to 43.02 Feet, the Content of the upper Part.

And

And extend the Compaſſes from 144 to half the Perpendicular 9.31, that Extent will reach from the Side 21.5 to 1.39 Feet, the Area of the Baſe; which, added to the other, makes 44.41 Feet, the Content of the Whole.



Let ABCDEFGH be a Pyramid, whoſe Baſe is Heptagon, each Side thereof being 15 Inches, and the Perpendicular of the Heptagon is 15.58 Inches, and the perpendicular Height of the Pyramid HI is 13.5 Feet; the Content, ſolid and ſuperficial, is required.

Multiply 15.58 (the Perpendicular) by 52.5 (half the Sum of the Sides of the Heptagon) and the Product is 817.95; which multiplied by 4.5, *viz.* $\frac{1}{3}$ of the Height, and the Product is 3680.775.

Then

Then divide this laſt Product by 144, and the Quotient is 25 56 Feet, the Content.

```
         15.58 the Heptagon's Perpend.
          52.5 the Half-Sum of the Sides.
        ------
          7790
         3116
        7790
      --------
       817.950
           4.5 a third Part of the Height.
      --------
       4089750
      3271800
    ----------
144)3680.7750(25.56 Solid Feet.
    288
    ---
     800
     720
     ---
      807
      720
      ---
       877
       864
       ---
        13
```

By Scale and Compaſſes:

Firſt, find a geometrical mean Proportional between 15.58 and 52.4 (as is before directed), which you will find to be 28.06; then extend the Compaſſes from 12 to 28.06, that Extent will reach from 4.5 (twice turned over) to 25.56 Feet.

To find the superficial Content.

Multiply the Height taken from the Middle of one of the Sides of the Base 162.75 Inches, by the Half-Sum of the Sides 32.5 Inches, and the Product is 8544.375; which divided by 144, the Quotient is 59.335 Feet, the Content of the upper Part.

```
     162.75                      144)817.95(5.6
       52.5                          -------
     ------                            979
      81375                            1155
     32550                             ----
    81375                                 3
    -------
144)8544.375(59.335 Feet.
    ----        5.68 Base add.
   1344      -------
     483     63.015 the whole Content.
      517
       855
      ----
       135
```

By Scale and Compasses.

Extend the Compasses from 144 to 162.75, that Extent will reach from 52.5 to 59.335 Feet.

And extend the Compasses from 144 to 15.58, the Perpendicular of the Heptagon, that Extent will reach from 5.25 to 5.68 Feet, the Content of the Base; which add to the former, the Sum is 65.015, the whole superficial Content.

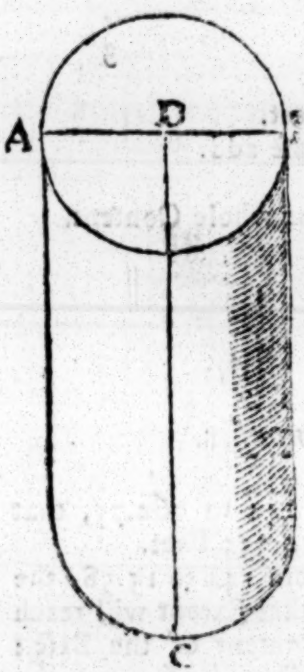
§ V.

§ V. *Of a* CYLINDER.

A Cylinder is a round Solid, having its Baſes circular, equal, and parallel, in Form of a Rolling-ſtone uſed in Gardens. To find the ſolid Content thereof, this is

The RULE.

Multiply the Area of the Baſe by the Length, and the Product is the ſolid Content.



Let ABC be a Cylinder, whoſe Diameter AB is 21.5 Inches, and the Length CD is 16 Feet; the ſolid Content is required.

Firſt, ſquare the Diameter 21.5, and it makes 462.25; which multiplied by .7854, and the Product is 363.05115. Then multiply this by 16, and the Product is 5808.8164. Divide this laſt Product by 144, and the Quotient is 40.34 Feet, the ſolid Content.

By Scale and Compaſſes.

Extend the Compaſſes from 13.54 to 21.5, the Diameter, that Extent (turned twice over from 16, the Length) will at laſt fall upon 40.34, the ſolid Content.

To

To find the ſuperficial Content.

Firſt (by Chap. I. Sect. IX. Prob. 2.), find the Circumference of the Baſe 67.54, which multiplied by 16, the Product is 1080.64; which divided by 12, the Quotient is 90.05 Feet, the curve Surface; to which add 504 Feet, the Sum of the two Baſes, and the Sum is 95.09 Feet, the whole ſuperficial Content.

67.54		363.05
16		2
40524		144)726.10(5.04
6754		610
12)1080.64	90.05 } add.	34
	5.04 }	
90.05	95.09	

By Scale and Compaſſes.

Extend the Compaſſes from 12 to 67.54 (the Circumference), that Extent will reach from 16 (the Length) to 91.05 Feet, the curve Surface.

And extend the Compaſſes from 12 to 21.5 (ihe Diameter), that Extent (turned twice from .7854) will at laſt fall upon 2.52 Feet, the Area of the Baſe; which doubled is 504; this, added to the curve Surface, makes 95.09 Feet, the whole ſuperficial Content.

Demonſtration. The ſolid Content of every Cylinder is found, by multiplying the Area of its Baſe into its Height, as aforeſaid: For every right Cylinder is only a round Priſm, being conſtituted of an infinite Series of equal Circles; that of its Baſe, or End, being one of the Terms, and its Height CD (in the former Figure) is the Number of all the Terms. Therefore the Area of its Baſe AB, being multiplied into CD,

CD, will be its Solidity (by Lemma I.). Let $D = AB$, $H = CD$.

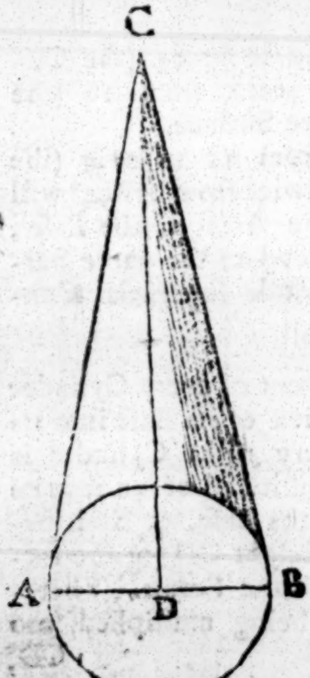
Then $.7854\ DD \times H =$ its Solidity,

§ VI. *Of a* CONE.

A Cone is a Solid, having a circular Baſe, and growing ſmaller and ſmaller, till it ends in a Point which is called the Vertex, and may be nearly repreſented by a Sugar-loaf. To find the Solidity thereof, this is

The RULE.

Multiply the Area of the Baſe by a third Part of the perpendicular Height, and the Product is the ſolid Content.



Let ABC be a Cone, the Diameter of whoſe Baſe AB is 26.5 Inches, and the Height of the Cone DC is 16.5 Feet: Firſt, ſquare the Diameter 26.5, and it is 702.25, which multiply by .7854, and the Product is 551.54715; which multiply by 5.5, and the Product is 3033.47825; which divided by 144, the Quotient is 21.07 *ferè*, the ſolid Content of the Cone.

26.5

```
          26.5 the Diameter.
          26.5
          ----
          1325
         1590
         530
        ------
        702.25 the Square.
         .7854
        ------
        280900
       351125
      561800
     491575
     ----------
     551.54|715 Area of the Base.
           5.5  a third Part of the Height.
    ----------
    275770
   275770
   ----------
144)3033.47|0(21.06 Feet, the Content.
    ---
     153
      947
     ---
      83
```

By Scale and Compasses.

Extend the Compasses from 13.54 to 26.5, the Diameter, that Extent turned twice over from 5.5 (a third Part of the Height, it will at last fall upon 21.06 Feet, the Content.

To find the superficial Content.

Multiply half the Circumference 41.626 by the slant Height AC 198.46, and the Product is 8261.09596; which divided by 144, the Quotient is 57.37 *ferè*, the curve Surface; to which add the Base, the Sum is 61.2, the superficial Content.

```
      41.626 half Circumference of the Baſe.
      198.46 the ſlant Height.
      ------
      249756
     166504
    333008
   374634
   41626
   -------------
144)8261.09596(57.37 Feet ferè.
    ----            3.83 the Baſe add.
   1061            -----
     530           61.20 the whole Content.
      989
   -------
144)551.54(3.83
    -----
    1195
      434
      ---
        2
```

By Scale and Compaſſes.

Extend the Compaſſes from 144 to 198.46, that Extent will reach from 41.626 to 57.37 Feet, the curve Surface.

And extend the Compaſſes from 12 to 26.5, the Diameter; that Extent, turned twice over from .7854, will at laſt fall upon 3.83 Feet, the Baſe; which added to 57.37, the Sum is 61.2 Feet, the ſuperficial Content.

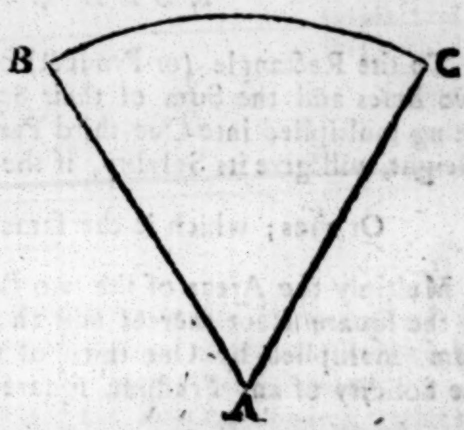
Demonſtration. Every Cone is the third Part of a Cylinder of equal Baſe and Altitude. The Truth of this may eaſily be conceived, by only conſidering, that a Cone is but a round Pyramid; and therefore it muſt needs have the ſame Ratio to its circumſcribing Cylinder, as the ſquare Pyramid hath to its circumſcribing

ſcribing Parallelopipedon; *viz.* as 1 to 3. However, to make it yet clearer, let it be farther conſidered, That

Every right Cone is conſtituted of an infinite Series of Circles, whoſe Diameters do continually increaſe in arithmetical Progreſſion, beginning at the Vertex, or Point C, the Area of its Baſe AB being the greateſt Term, and its perpendicular Height DC, the Number of all the Terms; therefore the Area of the Circle of the Baſe, multiplied by a third Part of the Altitude DC, will be the Sum of all the Series, equal to the Solidity of the Cone, by Lemma III.

The curve Superficies of every right Cone is equal to half the Rectangle of the Circumference of its Baſe into the Length of its Side.

For the curve Surface of every right Cone is equal to the Sector of a Circle, whoſe Arch BC is equal to the Periphery of the Baſe of the Cone, and Radius AB equal to the ſlant Side of the Cone: Which will appear very evident, if you cut a Piece of Paper in the Form of Sector of a Circle, as ABC, and bend the Sides AB and AC together, till they meet, and you will find it to form a right Cone.



I have omitted the Demonſtrations touching the Superficies of all the foregoing Solids, becauſe I thought it needleſs, they being all compoſed of Squares, Parallelograms, Triangles, *&c.* which Figures are all demonſtrated before. And if the Area of all ſuch Fi-

gures as compoſe the Solid, be found ſeverally, and added together, the Sum will be the ſuperficial Content of the Solid.

§ VII. *Of the Fruſtum of a* Pyramid.

A Fruſtum of a Pyramid is the remaining Part, when the Top is cut off by a Plane parallel to the Baſe. To find the ſolid Content thereof, there are ſeveral Rules.

RULE I.

To the Rectangle (or Product) of the Sides of the two Baſes add the Sum of their Squares; that Sum, being multiplied into One-third Part of the Fruſtum's Height, will give its Solidity, if the Baſes be ſquare.

Or thus; which is the ſame in Effect:

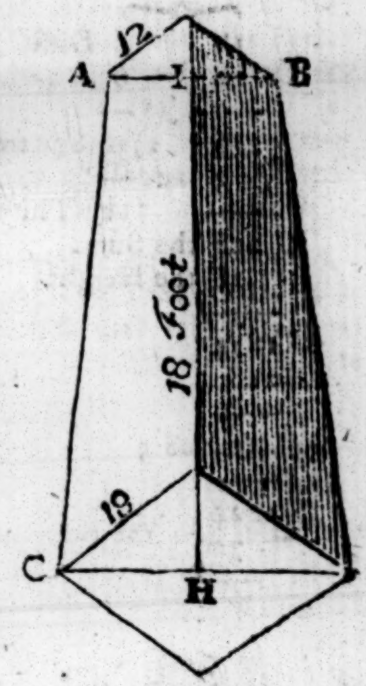
Multiply the Areas of the two Baſes together, and to the ſquare Root thereof add the two Areas; that Sum, multiplied by One-third of the Height, gives the Solidity of any Fruſtum, ſquare or multangled.

RULE II.

To the Rectangles of the Sides of the two Baſes, add one third Part of the Square of their Difference; that Sum, being multiplied into the Height, will produce the Solidity, if the Baſes be Squares: But if they be triangular or multangular, the ſaid Rectangle of the Sides, with the third Part of the Square of their Difference, will be the Square of a mean Side; and the ſquare Root thereof will be ſuch a mean Side as

as will reduce the tapering Solid to a Prifm equal thereunto.

Example. Let A B C D be the Fruftum of a fquare Pyramid, the Side of the greater Bafe 18 Inches, and the Side of the leffer 12 Inches, and the Height 18 Feet; the Solidity thereof is required.



Firft, multiply the two Sides together, 18 by 12, and the Product is 216, and the Difference of the Sides is 6, whofe Square is 36, a third Part thereof is 12, which added to 216, the Sum is 228 Inches, the Area of a mean Bafe; which multiplied by 18 Feet, the Length, the Product is 4104; this divided by 144, the Quotient is 28.5 Feet, the Content.

Or, by the firft Rule, thus: The Square of 18 is 324, and the Square of 12 is 144, and the Rectangle of 18 by 12 is 216; the Sum of thefe three is 684, which multiplied by 6, the Product is 4104; which divided by 144, the Quotient is 28.5 Feet, the fame as before.

See the Work both Ways.

18	6 Diff.	
12	6	
216	3)36 Square	
12 add		
	12 a Third.	
228 the Sum.		
18 the Height.		
1824		
228		
144)4104(28 5		
1224		
720		
...		

18	12
18	12
324 Sq	144 Sq.
144	
216	
684 the Sum.	
6 a 3d of the Height.	
144)4104(28.5 Feet.	
1224	
720	
...	

By Feet and Inches, thus:

	F.	I.	I.
Mult.	1	6	6
by	1		6
Prod.	1	6	3)36q.
add	0	1	
			12
Mult.	1	7	
by	18	0 Height.	
	18	0	
	9	0	
	1	6	
Cont.	28	6	

Or thus;

F.	I.	
2	3	Sq. of the greater.
1	6	the Rectangle.
1	0	Square of the less.
4	9	Trip. of a mean Ar.
6	0	a 3d of the Height.
28	6	

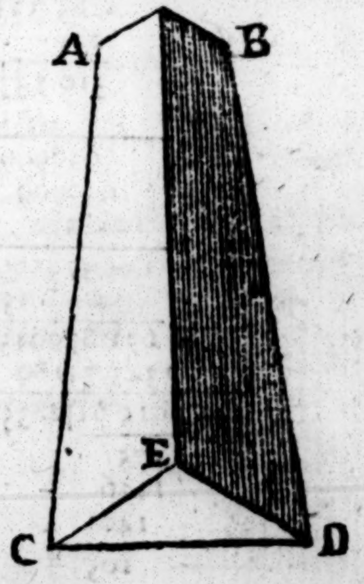
To

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 72, and the Perimeter of the leſſer Baſe is 48; add both the Perimeters together, the Sum is 120; the half thereof is 60; which multiplied by 18 Feet, the Product is 1080; this divided by 12, the Quotient is 90 Feet; to which add the two Baſes 2.25 Feet and 1 Foot; the Sum is 93.25 Feet, the whole ſuperficial Content.

18	12	18 the Height.
4	4	60
72	48	12)1080
48		90 Feet.
2)120		2.25 the greater Baſe.
60		1 the leſſer Baſe.
		93.25 the Sum.

Again: Let ABC be the Fruſtum of a triangular Pyramid, each Side of the greater Baſe 25 Inches, and each Side of the leſſer Baſe 9 Inches, and the Length 15 Feet; the ſolid Content thereof is required.



By the ſecond Rule, multiply 25 by 9, and the Product is 225; and the Difference between 25 and 9 is 16, which ſquared, makes 256; a third Part thereof is 85.333, which added to 225, the Sum is 310.333; and this multiplied by

433, the Product is 134 374, *&c.* which is the Area of a mean Baſe; and that multiplied by 15 Feet, the Length, the Product is 2015.610; which divided by 144, the Quotient is 13 99 Feet, the Solidity.

Or thus, by the latter Part of the firſt Rule: Find the Area of the greater Baſe, which you will find to be 270.625, and the Area of the leſſer Baſe will be 35 073; theſe two Areas multiplied together, the Product is 9491.630625; the Square Root thereof is 97.425; to which add the two Areas, and the Sum is 403.123; which multiplied by a third Part of the Length, 5, the Product is 2015.615; and that divided by 144, the Quotient is 13.99 Feet, as before.

See the Working of both.

```
           25      25
            9       9
Product   225      16  Diff.
                   16
                   96
                  16
               3)256    the Square.
                85.333  a third Part.
               225      add.
               310.333
                  .433  tabular Number, p. 89.
                930999
               930999
             1241332
             134.374189  mean Area.
                     15  Length.
             671.870945
            1343.74189
    144)2015 61|2835(13.99 Feet.
          575
          1436
          1401
           105
```

25

```
    25
    25
  ----
   125
   50
  ----
   625 Square.
   433
  ----
  1875
 1875
2500
  ----
270.625 Area.
```

```
 9
 9
--
81 Square.
```

```
     433
      81
    ----
     433
   3464
  ------
  35.073

   270.625
    35.073
   -------
    811875
  1894375
 13531250
 811875
 ------------
 9491.630625
```

```
   . .
   9491.630625(97.425
   81
   ---
187)1391
    1309
   -----
1944)8263
     7776
    -----
19482)48706
      38964
     ------
194845)974225
       974225
      -------
      ......
```

270.625

270.625	greater Area.
97.425	the mean Proportional.
35.073	the leſſer Area.
403.123	the Triple of a mean Area.
5	a third Part of the Height.

144)2015.615(13.99 Feet, the Solidity.

575
1436
1401

105

In finding the Area of the triangular Baſe, I multiply by .433, becauſe that is the Area of the equilateral Triangle, when the Side thereof is 1. A Table of the Areas, or Multipliers, for finding the Areas of Polygons, you'll find in *p.* 89.

Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

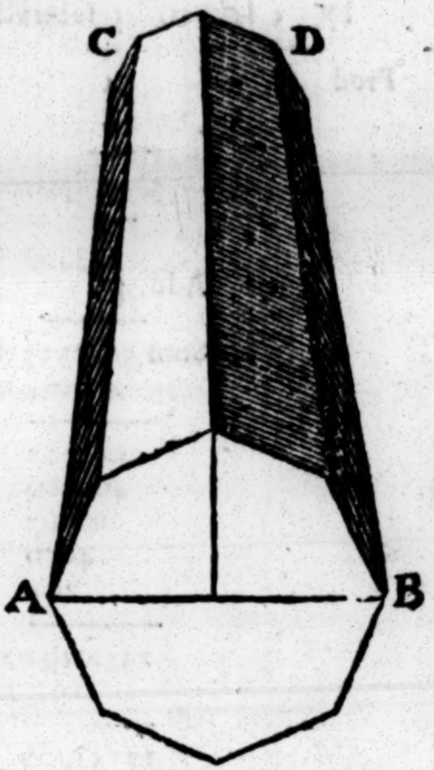
To find the ſuperficial Content.

The Perimeter of the greater Baſe is 75, and the Perimeter of the leſſer Baſe is 27; the Sum of both is 102, and the half Sum is 51; which multiplied by 15 Feet, the Product is 765; which divided by 12, the Quotient is 63.75; to which add the Sum of the two Baſes 2.12 Feet, and the Sum is 65.87 Feet, the whole ſuperficial Content.

Note, That 51 ſhould have been multiplied by the ſlant Height, but the Difference it would make is but .06 of a Foot, which is inconſiderable.

Again:

Again: Suppoſe ABCD to be the Fruſtum of a Pyramid, having an octagonal Baſe, each Side thereof being 9 Inches, and each Side of the leſſer Baſe 5 Inches, and the Length, or Height, 10.5 Feet; the Solidity is required.



By the ſecond Rule multiply the greater Side 9 by the leſſer Side 5, and the Product is 45; then the Difference between 9 and 5 is 4; which ſquared makes 16; a third Part thereof is 5.3333, which added to 45, the Sum is 50.3333; multiply this laſt by the Number in the Table 4.8284, and the Product is 243.0292, the Area of a mean Baſe; which multiplied by the Height 10.5 Feet, the Product is 2551.8066; then divide this laſt Product by 144, and the Quotient is 17.72 Feet, the ſolid Content.

See the Work.

Mult.

Mult. 9 Inches. 9 from the greater Side.
by 5 Inches. 5 subtract the lesser.

Prod. 45 4
4

3)16 squared.

5.3333 a third Part.
Add 45

Sum 50.3333 the Sq. of a mean Side.
4.8284 tabular Number, *p.* 89.

2013332
402666
10067
4026
201

243.0292 a mean Area.
10.5 the Height.

12151460
2430292

144)2551.80|660(17.72
144

1111
1008

1038
1008

300
288

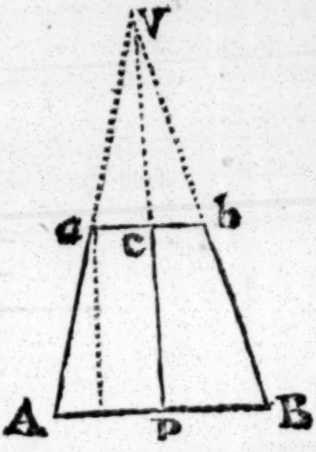
12

To

To find the superficial Content.

The Perimeter of the greater Base is 72, and the Perimeter of the lesser Base is 40, and their Sum is 112; the Half thereof is 56, which multiplied by the Height 10 5 Feet, and the Product is 588; which divided by 12, the Quotient is 49 Feet; to which add the Sum of the two Bases, 3.55, and the Sum is 52.55 Feet, the whole superficial Content.

Demonstration. From the Rules delivered in the IVth and VIth Sections, the two foregoing Rules may easily be demonstrated.



Suppose a square Pyramid, ABV, to be cut by a Plane at *a* b, parallel to its Base AB and it were required to find the Solidity of the Frustum, or Part *a* b AB. Let there be given

$D = BA$, the Side of the greater Base.
$d = ba$, the Side of the lesser Base.
$H = CP$, the perpendicular Height.

First, | 1 | $D - d : H :: d : \frac{dH}{D-d} = VC$ by the Figure.

Then | 2 | $DD \times \frac{H + VC}{3} =$ the whole Pyramid BVA, by Section the IVth.

And | 3 | $dd \times \frac{1}{3}VC =$ the Pyramid *a* Vb cut off.

Then, in the 2d and 3d Steps, if, inftead of VC, you take $\frac{dH}{D-d}$ equal to it by the firft Step, it will be,

viz. 1. 2.	4	$\frac{DDDH}{3D-3d}$ = the whole Pyramid BVA ;
and 1. 3.	5	$\frac{ddd H}{3D-3d}$ = the Pyramid *a* V b.
4.—5.	6	$\frac{DDDH-dddH}{3D-3d}$ = the Fruftum *a* b A B.

And by dividing DDD—ddd by D—d, and then multiplying the Quotient by $\frac{1}{3}$ H, the laft Step will be reduced to $DD+Dd+dd : \times\frac{1}{3}H$ = the Fruftum *a* b A B ; which, in Words, is thus :

To the Rectangle of the Sides of the two Bafes add the Sum of their Squares; that Sum being multiplied into One-third of the Fruftum's Height, will give its Solidity ; which is the fame as the firft Rule of this Section.

See the Work of the Divifion.

$$
\begin{array}{rl}
D-d) & DDD-ddd\ (DD+Dd+dd \\
 & DDD-DDd \\
\hline
 & \quad DDd-ddd \\
 & \quad DDd-Ddd \\
\hline
 & \qquad Ddd-ddd \\
 & \qquad Ddd-ddd \\
\hline
 & \qquad\quad 0 \quad 0
\end{array}
$$

The

The ſame Reaſon will hold good for all Fruſtums of Pyramids or Cones, whether the Baſe be triangular or multangular, becauſe the Squares of the Sides of any Figure, or the Squares of the Diameters of Circles, are proportional to the Area; which proves the latter Part of the ſaid firſt Rule.

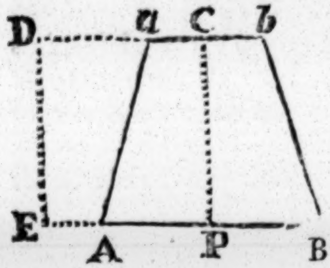
Again, to prove the ſecond Rule.

Suppoſe	1	$x=D-d$. And $F=$the Fruſtum.
then	2	$DD+Dd+dd=\frac{3F}{H}$ by the laſt.
1 ⊙ 2	3	$xx=DD-2Dd+dd$.
2 — 3	4	$3Dd=\frac{3F}{H}-xx$.
4 — 3	5	$Dd=\frac{F}{H}=\frac{1}{3}xx$. Or $Dd+\frac{1}{3}xx=\frac{F}{H}$
5 × H	6	$.Dd+\frac{1}{3}xx: \times H=F$, the Fruſtum *a* b A b.

Which, in Words, is thus:

To the Rectangle of the Sides of the two Baſes add one-third Part of the Square of the Difference of the ſaid Sides, and multiply the Sum of the Height of the Fruſtum, the Product is the Solidity of the Fruſtum.

The ſuperficial Contents of Fruſtums (all but the Baſes) are compoſed of Trapeziums, ſo many as the Fruſtum has Sides. As the ſquare Fruſtum *a*bAB, in the laſt Figure is compoſed of four Trapeziums, having the two upper, and alſo the two lower Angles equal; if therefore the Trapezium *a*bAB be cut in two by the Line CP, and the two Pieces laid together, the Line bB upon the Line *a*A, the narrow End of the one to the broad End of the other, it will form a right-angled Parallelogram, as is plain by the Figure an-



nexed; the Parallelogram DCEP being equal to the Trapezium *ab*AB; becauſe the Side D*a* is equal to PB, and EA is equal to *a*C. Therefore, to find the Area of the Trapezium, add half the Side *ab* to half the Side AB, and it makes DC or EP; which multiplied by the Height PC, the Product is the Area of the Parallelogram DCEP, equal to the Trapezium *ab*AB; then, if that be multiplied by the Number of Trapeziums, the Product will be the ſuperficial Content of the Fruſtum, wanting the Baſes. Or, if the whole Perimeter of the greater Baſe be added to the Perimeter of the leſſer Baſe, and half the Sum multiplied by the Height, the Product will be the ſuperficial Content of all the Trapeziums at once.

Note, That half the Sum of the Perimeters ſhould be multiplied by the ſlant Height, up the Middle of one of the Trapeziums; but in the foregoing Examples I have multiplied by the perpendicular Height becauſe the Difference is very inconſiderable.

§ VIII. *Of the Fruſtum of a* Cone.

A Fruſtum of a Cone, is that Part which remains when the top End is cut off by a Plane parallel of the Baſe. To find the ſolid Content, the Rules are the ſame in effect as for the Fruſtum of a Pyramid.

RULE I.

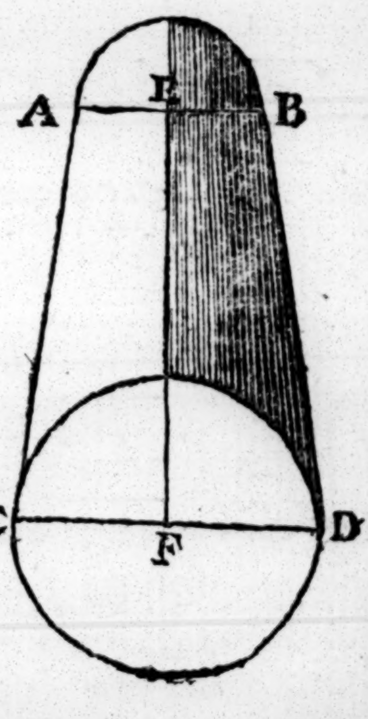
To the Rectangle of the Diameter of the two Baſes add the Squares of the ſaid Diameters, and multiply the

the Sum by .7854, the Product will be the Triple of a mean Area; which multiplied by $\frac{1}{3}$ of the perpendicular Height, that Product will be the solid Content.

Or thus: Multiply the Areas of the greater and lesser Bases together, and out of the Product extract the Square Root, and add the two Areas and Square Root together, and multiply the Sum by One-third of the perpendicular Height, the Product is the solid Content.

RULE II.

To the Rectangle of the greater and lesser Diameters, add one-third Part of the Square of their Difference, and multiply the Sum by .7854, the Product is a mean Area; which multiplied by the perpendicular Height, the Product is the Solidity.



Example. Let ABCD be the Frustum of a Cone, whose greater Diameter CD is 18 Inches, and the lesser Diameter AB 9 Inches, and the Length 14.25 Feet, the solid Content is required.

Multiply 18 by 9, and the Product is 162, and the Difference between 18 and 9 is 9, whose Square is 81; a third Part is 67, which add to 162, the Sum is 189; this multiplied by .7854, the Product is 148.44; which divided by 144, the Quotient is 1.03

 Feet,

Feet, the Area of a mean Baſe; which multiplied by 14.25 Feet, the Height, the Product is 14.6775 Feet, the ſolid Content.

Or thus, by the firſt Rule.

The Square of 18 (the greater Diameter) is 324, and the Square of 9 (the leſſer Diameter) is 81, and the Rectangle, or Product of 18 by 9, is 162; the Sum of theſe three is 567, which multiplied by .7854, the Product is 445.3218; which divided by 144, the Quotient is 3.09 Feet, the triple Area of a mean Baſe; this multiplied by 4.75 Feet (a third Part of the Height), and the Product is 14.6775 Feet, the Solidity, the ſame as before.

See the Work.

	18	18	from	.7854
	8	9	ſubtr.	189
	162	9	Rem.	70686
Add	27	9		62832
				7854
Sum	189	3)81	Square.	
				144)148.4406(1.03
		27	a Third.	144
Height		14.25	Feet.	444
Area Baſe		1.03	Feet.	432
		4275		12
		14250		
Solid Content		14.6775	Feet.	

```
324 the Square of 18.
162 the Rectangle.
 81 the Square of 9.
———
567 the triple Square of a mean Diameter.

      .7854
        567
      ——————
      54978
     47124
    39270
    ———————
144)445.3218        (3.09
    ——————            4.75
    1332             ——————
    ————              1545
      36             2163
                    1236
                    ——————
      The Solidity  14.6775
```

To find the ſuperficial Content.

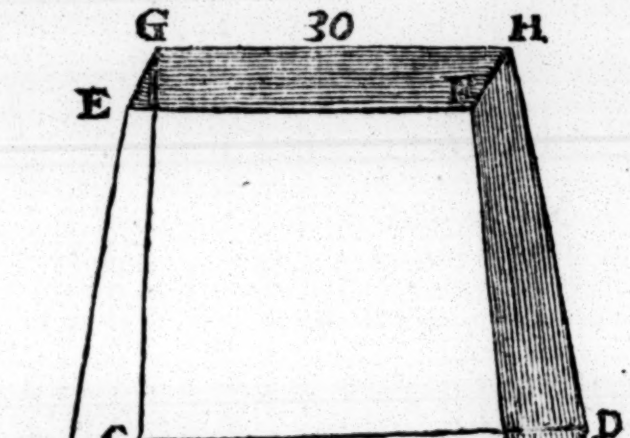
By Chap. I. Sect. IX. Problem 2. you will find the Circumference of the greater Baſe to be 56.5488, and of the leſſer Baſe 28.2744; the Sum of both is 84.8232; the Half-ſum is 42.4116; which multiplied by 14.25 Feet, and the Product is 604.36, &c. which divided by 12, the Quotient is 50.36 Feet, the curve Surface; to which add the Sum of the two Baſes, 2.21 Feet, the Sum is 52.75 Feet, the whole ſuperficial Content.

§ IX. *To meaſure the Fruſtum of a rectangled* Pyramid, *called a* PRISMOID, *whoſe Baſes are parallel one to another, but diſproportional.*

The RULE.

TO the greateſt Length add half the leſſer Length, and multiply the Sum by the Breadth of the greater Baſe, and reſerve the Product.

Then, to the leſſer Length, add half the greater Length, and multiply the Sum by the Breadth or the leſſer Baſe, and add this Product to the other Product reſerved, and multiply that Sum by a third Part of the Height, and the Product is the ſolid Content.



Example.

Example. Let ABCDEFGH be a Priſmoid given, the Length of the greater Baſe AB 38 Inches, and its Breadth AC 16 Inches; and the Length of the leſſer Baſe EF is 30 Inches, and its Breadth 12 Inches, and the Height 6 Feet; the ſolid Content is required.

To the greater Length AB 38, add half EF the leſſer Length 15, the Sum is 53; which multiplied by 16, the greater Breadth, and the Product is 848; which reſerve.

Again, to EF 30, add half AB 19, and the Sum is 49; which multiplied by 12 (the leſſer Breadth EG), the Product is 588; to which add 848 (the reſerved Product), and the Sum is 1436; which multiplied by 2 (a third Part of the Height), and the Product is 2872; divide this Product by 144, and the Quotient is 19.94 Feet, the ſolid Content.

$38 = AB$
$15 = \frac{1}{2}EF$

53
$16 = AC$

318
53

848
588

1436
$2 =$ a third Part of the Height.

144)1872(19.94 Feet, the Content.

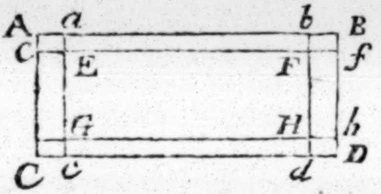
1432
1360
640

64

$30 = EF$
$19 = \frac{1}{2}AB$
49
$12 = EG$

588

To prove this Rule, Let us suppose the Solid cut into Pieces, so as to make it capable of being measured by the foregoing Rules; thus: Let ABCD represent the greater Base, and EFGH the lesser Base; and let the Solid be supposed to be cut thro' by the Lines, *ac*, *bd*, and *ef*, *gh*, from the Top to the Bottom; so will there be a Parallelopipedon, having its Bases equal to the lesser Base EFGH, and its Height 6 Feet, equal to the Height of the Solid: Multiply 30 (the Length of the Base by 12, the Breadth thereof), and the Product is 360; which multiplied by the Height 6 Feet, and the Product is 2160. Then there are two Wedge-like Pieces, whose Bases are *ab*EF, and GH*cd*; if these two Pieces be laid together, the thick End of one to the thin End of the other, they will compose a rectangled Parallelopipedon; which to measure, multiply the Length of the Base 30 by its Breadth 2, and the Product is 60; which multiplied by 6 (the Height), the Product is 360. Then there are two other Wedge-like Pieces, whose Bases are *e*E*g*G, and *f*FH*h*; these two laid together will compose a rectangled Parallelopipedon: To measure this, multiply the Length of the Base 12 by the Breadth 4, the Product is 48; which multiplied by 6 (the Height), the Product is 288. And lastly, there are four rectangled Pyramids, at each Corner one; which to measure, multiply the Length of one of the Bases 4 by its Breadth 2, the Product is 8; which multiplied by 2 (a third Part of the Height) the Product is 16; and that multiplied by 4 (because there are four of them), the Product is 64. Then add all these together, and the Sum is 2872, and divide by 144, the Quotient is 19.94 Feet, the same as before; which shews the Rule to be true.



See

See the Work.

12	30	12	4
30	2	4	2
360	60	48	8
6	6	6	2
2160	360	288	16
360			4
288			64
64			

144)2872(19.94 Feet, the whole Content.

1432
1360
640

64

To find the ſuperficial Content.

Half the Perimeter of the greater Baſe is 54, and half the Perimeter of the leſſer Baſe is 42, which added together, the Sum is 96; which multiplied by 6 (the Height), the Product is 576: Divide this Product by 12, the Quotient is 48 Feet; to which add the Sum of the two Baſes 6.72 Feet, and the Sum is 54.72 Feet, the whole ſuperficial Content.

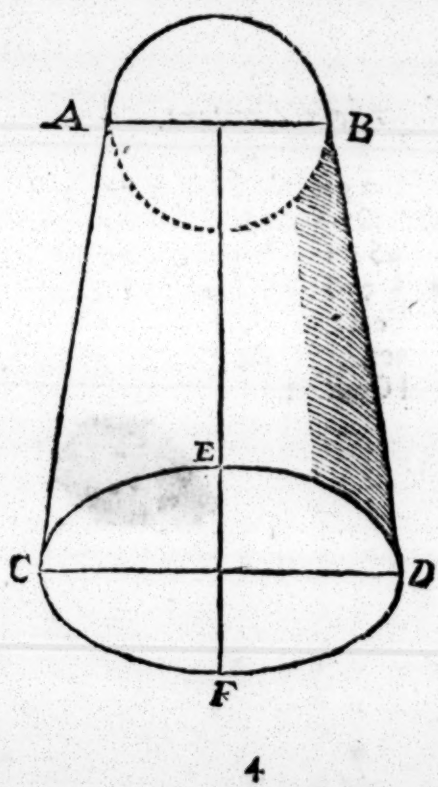
To

To meaſure a CYLINDROID; *that is, a Fruſtum of a* Cone, *having its Baſes parallel to each other, but unlike.*

The RULE.

TO the longeſt Diameter of the greater Baſe, add half the longeſt Diameter of the leſſer Baſe, and multiply the Sum by the ſhorteſt Diameter of the greater Baſe, and reſerve the Product.

Then, to the longeſt Diameter of the leſſer Baſe, add half the longeſt Diameter of the greater Baſe, and multiply the Sum by the ſhorteſt Diameter of the leſſer Baſe, and add the Product of the former reſerved Sum, and that Sum will be the triple Square of a mean Diameter; which multiplied by .7854, and that Product multiplied by a third Part of the Height, the Product is the ſolid Content.



Exam. Let ABCD be a Cylindroid, whoſe Bottom-baſe is an Oval, the tranſverſe Diameter being 44 Inches; and the conjugate Diameter 14 Inches; and the upper Baſe is a Circle, whoſe Diameter is 26 Inches; and the Height of the Fruſtum is 9 Feet; the Solidity is required.

To 44 (the greater Diameter of the lower Baſe) add 13 (half the

the Diameter of the greater Baſe, the Sum is 57; which multiplied by 14 (the conjugate Diameter of the greater Baſe, the Product is 798; which reſerve. Then to 26 (the Diameter of the leſſer Baſe) add 22 (half the traverſe Diameter of the greater Baſe), and the Sum is 48; which multiplied by 26 (the Diameter of the leſſer Baſe), the Product is 1248; to which add the former reſerved Product, the Sum is 2046; which multiplied by .7854, the Product is 1606.9284; which multiplied by 3 (a third Part of the Height), the Product is 4820.7852; which divided by 144, the Quotient is 33.47 Feet, the ſolid Content. See the Work.

```
44=CD                        26=AB
13=half AB                   22=half CD
——                           ——
57 Sum.                      48 Sum.
14=EF                        26=AB
————                         ————
228                           288
57                            96
————                         ————
798 Product reſerved.        1248
                              798 add.
                             ————
                             2046
                             .7854
                             ————
                             8184
                            10230
                           16368
                          14322
                         ———————
                         1606.9284
                                 3
                    ——————————————
                144)4820.78|52(33.47
                    ———
                     500
                      687
                      1118
                      ————
                       110
```

R

This

This Rule being the same as that in the last Section, the Proof of that may serve as a sufficient Proof of this, if what has been before written be well considered.

To find the superficial Content.

To the Periphery of the Ellipsis 97.41, add the Periphery of the Circle 81.68, and the Sum is 179.09; the half thereof 89.545, multiplied by 9, the Product is 805.905; which divided by 12, the Quotient is 67.16 Feet, the curve Surface: Then the Area of the Ellipsis is 3.36 Feet, and the Area of the Circle is 3.68 Feet; both which added to the curve Surface, the Sum is 74.2 Feet, the whole superficial Content.

§ XI. *Of a* SPHERE *or* GLOBE.

A Sphere, or Globe, is a round solid Body, every Part of whose Surface is equally distant from a Point within it, called its Center; and it may be conceived to be formed by the Revolution of a Semicircle round its Diameter. To find its Solidity, this is

The RULE.

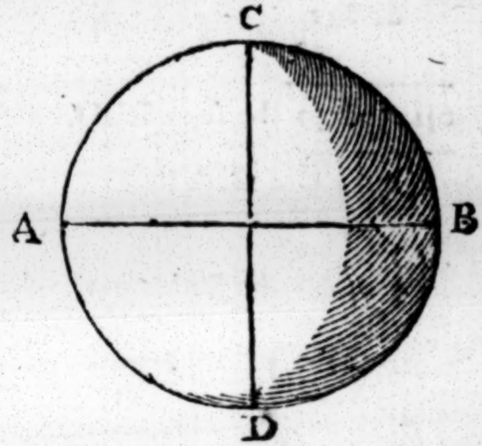
1. Multiply the Axis, or Diameter, into the Circumference, the Product is the superficial Content; which multiplied by a sixth Part of the Axis, the Product is the Solidity.

2. Or thus: As 21 is to 11, so is the Cube of the Axis to the solid Content.

3. Or, As 1 is to .5236, so is the Cube of the Axis to the solid Content.

Example

Example. Let ABCD be a Globe whoſe Axis is 20 Inches, then the Circumf. will be 62.832 : Then, by the firſt Rule, multiply the Circumference by the Axis, and the Product will be 1256.64, which is the ſuperficial Content in Inches; take a ſixth Part thereof, which is 209.44 (becauſe an exact ſixth Part of 20 cannot be taken), multiply that ſixth Part by 20 (the Axis), and the Product is 4188.8, the Solidity in Inches. Or, if you multiply the ſuperficial Content by the Axis, and take a ſixth Part of the Product, the Anſwer will be the ſame.



Or thus, by the ſecond Rule :

The Cube of the Axis is 8000 ; which multiplied by 11, the Product is 88000; which divided by 21 the Quotient is 4190.47, the Solidity.

Or, by the third Rule:

If the Cube of the Axis be multiplied by .5236, the Product is 4188.8, the Solidity, the ſame as by the firſt Way. If you divide 4188.8 by 1728, the Quotient is 2.424 Feet.

See the Work.

```
         62.832
             20
     ─────────────
     6)1256.640 the fuperficial Content.
     ─────────────
        209 44  a fixth Part.
            20
        ──────
        4188.80 the Solidity in Inches.

           21 : 11 : : 8000
                          11
                    ─────────
                 21)88000(4190.47 the Content.
                    ─────────
                      40
                      190
                         100
                          160
                         ─────
                           13

     1 : .5236 : : 8000
               8000
     ─────────────────
  1728)4188 8000(2.424 Feet, the Solidity.
     ─────────
        7328
         4160
          7040
          ─────
           128
```

Note, If the Axis of a Globe be 1, the Solidity will be .5236; and if the Circumference be 1, the Solidity will be .016887.

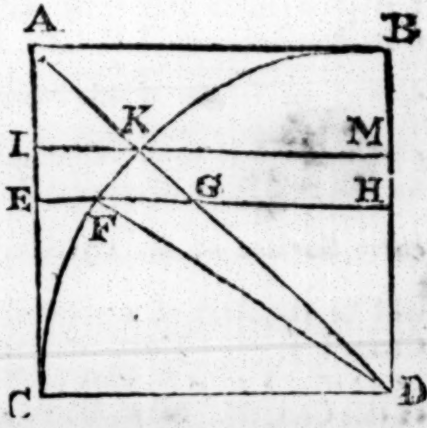
By Scale and Compaſſes.

Extend the Compaſſes from 1 to 20 (the Axis), that Extent (turned three times over from .5236), will at the laſt fall upon 4188.8, the ſolid Content in Inches: Or, extend the Compaſſes from 1728 to 8000 (the Cube of the Axis), that Extent will reach from .5236 to 2.424, the ſolid Content in Feet.

Extend the Compaſſes from 1 to 20 (the Axis), that Extent (turned twice over from 3.1416), will at laſt fall upon 1256.64, the ſuperficial Content in Inches: Or, extend the Compaſſes from 144 to 400 (the Square of the Axis), that Extent will reach from 3.1416 to 8.72, the ſuperficial Content in Feet.

Demonſtration. Every Sphere is equal to a Cone, whoſe perpendicular Axis is the Radius of the Sphere, and its Baſe a Plane equal to all the Surface of it.

For you may conceive the Sphere to conſiſt of an infinite Number of Cones, whoſe Baſes, taken altogether, compoſe the Surface, and whoſe Vertexes meet altogether in the Center of the Sphere: Hence the Solidity of the Sphere will be gained, by multiplying its Surface by $\frac{1}{3}$ of its Radius.



Let the Square ABCD, the Quadrant CBD, and the right-angled Triangle ABD, be ſuppoſed all three to revolve round the Line BD as an Axis: Then will the Square generate a Cylinder, the Quadrant a Hemiſphere, and the Triangle a Cone, all of the ſame Baſe and Altitude.

Then the Square of EH ($=\square$FD) $=\square$FH$+$ $\square$DH (but DH$=$GH). And since Circles are as the Squares of their Diameters (by *Euclid* 12. 2.), the Circle made by the Revolution of EH must be equal to both the Circles made by the Motions of FH and GH.

If you take the Circle made by the Revolution of FH from both, there will remain the Circle made by the Motion of GH, equal to the Ring described by the Motion of EF. And thus it will always be, wherever you draw the Line EH or IM, *&c.*

Therefore the Aggregate, or Sum, of all the Rings, made by the Revolution of the EF's, must be equal to that of all the Circles made by the Motion of the GH's; *i. e.* the Dish-like Solid, formed by the revolving Rings, will be equal to the Cone, formed by the Revolution of the GH's, which are the Elements of the Triangle ABD; that is, the Dish-like Solid will be as the Cone, $\frac{1}{3}$ of the circumscribing Cylinder, and consequently the Hemisphere must be $\frac{2}{3}$ of it: Wherefore the Sphere is $\frac{2}{3}$ of the circumscribing Cylinder.

Let the Radius of the Sphere be r$=$CD, then the Diameter will be 2 r, let the Surface of the Sphere, generated by the revolving Semicircle, be called S, and that of the Cylinder, formed by the Revolution of 2 AC$=$2 r$=$Diameter, be called s. Wherefore in what was just now proved, the Expression for the Solidity of the Sphere in this Notation will be $\frac{1\,rS}{3}$; and putting c equal to the Circumference of the Base, or for the Periphery of a great Circle of the Sphere, the curve Surface of the Cylinder will be 2 r c, also $\frac{rc}{2}$ will be the Area of a great Circle (by Sect. IX. of Chap. I. Prob. 1.) and this multiplied by 21, makes 11 c; which is the Solidity of the Cylinder, by Sect. V. of this Chapter. Now, since s was put equal to 2 rc$=$ the

the curve Surface of the Cylinder $\frac{ſr}{2}$ (by ſubſtituting ſ for 2 r c) will be alſo = the Solidity of the Cylinder. Now, ſince the Sphere is = $\frac{2}{3}$ of the Cylinder, $\frac{rS}{3} = \frac{2 \times ſr}{3 \times 2}$; that is, $\frac{rS}{3} = \frac{2ſr}{6} = \frac{ſr}{3}$. Wherefore r S = r ſ, that is, dividing by r, S = ſ; or the Surface of the Sphere is equal to the curve Surface of the Cylinder, but the curve Surface of the Cylinder was 2 r c.

Whereſore, to find the Area of the Surface of either Sphere or Cylinder, you muſt multiply the Diameter (= 2 r) by the Circumference of a great Circle of the Sphere, or by the Periphery of the Baſe. From this Notation alſo $\frac{rc}{2}$, the Area of a great Circle of the Sphere is plainly $\frac{1}{4}$ of 2 r c, the Surface of the Sphere; that is, the Surface of the Square is Quadruple of the Area of the greateſt Circle of it.

Whereſore, to 2 r c, the convex Surface of the Cylinder, add r c, equal to the Area of both its Baſes, you will have 3 r c; which ſhews you, that the Surface of the Cylinder (including its Caſes) is to the Surface of the Sphere as 3 to 2; or that the Sphere is $\frac{2}{3}$ of the circumſcribing Cylinder, in Area as well as Solidity.

Or you may prove the Sphere to be $\frac{2}{3}$ of the Cylinder of the ſame Baſe and Altitude, by Lemma VI. aforegoing thus:

Let AGB repreſent the Hemiſphere, and AIKB half the Cylinder; then, if the Semidiameter GH be divided into ſix equal Parts, and Lines drawn parallel to AB, the Diameter, the Squares of the Semichords, ab, cd, ef, &c. will

 be

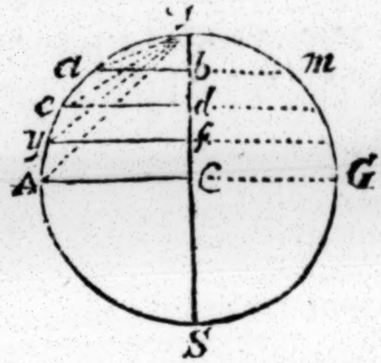
be a Series of Numbers, whoſe greateſt Term AH is a ſquare Number, the other differing by odd Numbers; that is, AH is 36, kl 35, gh 32, ef 27, cd 20, ab 11: But an infinite Series of ſuch Numbers are in Proportion to the infinite Number of Terms equal to the greateſt as 2 to 3. And becauſe the Hemiſphere is compoſed of an infinite Number of Circles, whoſe Diameters are the Chords of the Semicircle; and the Half-cylinder is compoſed of an infinite Number of Circles, whoſe Diameters are all equal to the Diameter of the Semicircles AB; therefore the Hemiſphere is in Proportion to the Half-cylinder as 2 to 3; and conſequently the whole Sphere bears the ſame Proportion to the whole Cylinder.

That the Superficies of every Sphere (or Globe) is equal to four times the Area of its greateſt Circle, is thus proved:

The Solidity of the Sphere is conſtituted of an infinite Number of parallel Circles (as is aforeſaid); conſequently the Superficies of the Sphere will be compoſed of the Peripheries of thoſe Circles which conſtitute its Solidity.

Note, In the following Demonſtration, ⊙ ſignifies any Circle in general; and if any two Letters be joined to it, thus, ⊙ AB, &c. then it denotes the Area of ſuch a Circle as thoſe two Letters repreſent the Radius of.

Let



Let $D = TS$, the Axis of any Sphere; then, according to the Property of a Circle, it

will be	1	$\overline{D - Tb} \times Tb = \square ab$;
that is,	2	$D \times Tb - \square Tb = \square ab$;
therefore	3	$D \times Tb = \square aT$.
For	4	$\square AB + \square Tb = \square aT$ (*Eucl.* 1. 47.)
and	5	$D \times DT = \square eT$.
	6	$D \times Tf = \square yT$.

Hence it is evident, that the Series $\square aT$, $\square eT$, $\square yT$, *&c.* are in the ſame Ratio with Tb, Td, Tf, *&c. viz.* in arithmetical Progreſſion: Whence it follows, that the $\odot$ *a*T = to the Sum of all the Circles Peripheries between T and b.

And $\odot$ eT = the Sum of all the Circles Peripheries between T and d, *&c.*

Conſequently, that the $\odot$ AT = the Sum of all the Circles Peripheries, included between T and C; that is, $\odot$ AT = the Superficies of the Hemiſphere.

And becauſe $\square$ AC + $\square$ TC = $\square$ AT, and $\square$ AC is equal to $\square$ TC; therefore $\odot$ AT = 2 $\odot$ AC, is the Superficies of the Hemiſphere.

Conſequently, 4 $\odot$ AC will be the Superficies of the whole Sphere. Which was to be proved.

Scholium.

Scholium.

From the Method here uſed in proving the whole Superficies, it will be eaſy to find the curve Superficies of any Fruſtum, or Part of a Sphere, that is cut off by a Right Line, or Plane; *viz.* ſuch as the Fruſtum aTm in the laſt Scheme, whoſe curve Superficies is ⊙aT. as above. Therefore (becauſe □ ab + □ Tb = aT) it will be ⊙ ab + ⊙ Tb = the curve Superficies of that Fruſtum.

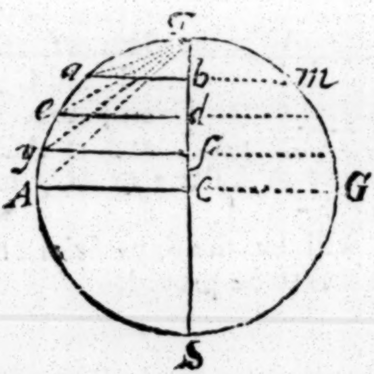
But if the Axis TS, and the Height Tb of the Fruſtum, are given, then it will be TS × Tb = □ aT, as in the third Step above; which gives the Proportion or Theorem following; *viz.*

As the Axis of the Sphere is to the whole Superficies of the Sphere; ſo is the Height of any Fruſtum to its curve Superficies.

To which if there be added the Area of the Fruſtum's Baſe, the Sum will be the whole Superficies of the Fruſtum.

That the Solidity of every Sphere is Two-thirds of its circumſcribing Cylinder, *may be thus proved.*

According to the Work above, it appears, that ⊙ ab, ⊙ ed, ⊙ yf, *&c.* do conſtitute the Solidity of the Sphere; and that □ aT, □ eT, □ yT, *&c.* are a Series of Terms in arithmetical Progreſſion, □ AT being the greateſt Term, and TC the Number of Terms; therefore ⊙ AT × $\frac{1}{2}$TC = the Sum of all the Series, by Lemma 2.



And becauſe □ aT − □ Tb = □ ab. □ aT − □ Td = □ ed. □ yT − □ Tf = □ yf. □ AT − □ TC = □ AC, *&c.* wherein

wherein $\square$ Tb, $\square$ Td, $\square$ Tf, *&c.* are a Series of Squares, whoſe Roots Tb, Td, Tf, are in arithmetical Progreſſion; $\square$ TC being the greateſt Term, and TC the Number of Terms; therefore $\odot TC \times \frac{1}{3} TC$ = the Sum of all the Series, by Lemma III.

Conſequently, $\odot AT \times \frac{1}{2} TC - \odot TC \times \frac{1}{3} TC =$ the Sum of all the Series $\odot$ ab, $\odot$ ed, $\odot$ yf, *&c.* which conſtitute the Solidity of the Half-ſphere ATG. Put $D = 2TC$, the Axis of the Sphere; then $\frac{1}{3} D = \frac{1}{2} TC$, and $\frac{1}{6} P = \frac{1}{3} TC$. And becauſe $\square AT = 2 \square TC$, therefore $\odot AT = 2 \odot TC = 1.5708\ DD$; and $1.5708\ DD \times \frac{1}{4} D = 0.3927\ DDD$.

Again; $\odot TC \times \frac{1}{3} TC = 0.7854\ DD \times \frac{1}{6} D = .1309\ DDD$, then $0.3627\ DDD - 0.1309\ DDD = 0.2618\ DDD$, the Solidity of the Half-ſphere.

Conſequently, $0.2618\ DDD \times 2 = .5236\ DDD$ will be the ſolid Content of the whole Sphere, which is equal to $\frac{2}{3}$ of the Cylinder; the Diameter of whoſe Baſe and Height, is $= D$.

For $0.7854\ DDD =$ the Solidity of the Cylinder, by Sect. V. But $\frac{2}{3}$ of $0.7854\ DDD = 0.5236\ DDD$, as before.

Scholium.

From this Demonſtration it will be eaſy to deduce or raiſe Theorems for finding the ſolid Content of any Fruſtum of a Sphere, as Tm in the laſt Figure.

For we there ſuppoſe the Fruſtum aTm to be conſtituted of an infinite Series of Circles, which have the ſame Ratio with all thoſe Circles that conſtitute the Half-ſphere.

Therefore it follows, that $\odot aT \times \frac{1}{2} TB$; — $\odot bT \times \frac{1}{3} Tb$ will be the Sum of all the Circles intercepted between T and b; conſequently it will be the Solidity of that Fruſtum.

And, becauſe $\square ab + \square Tb = aT$; therefore $\odot ab + \odot Tb \times \frac{1}{2} Tb$:— $\odot Tb \times \frac{1}{3} Tb =$ the Solidity. Let $c = ab$ half the Diameter of the Fruſtum's Baſe $h = Tb$ its Height; and $S =$ the Solidity of the Fruſtum. Then $\odot ab = 3.141600$, and $\odot Tb = 3.1416hh$; conſequently,

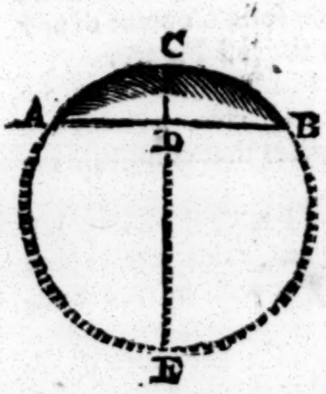
ſequently, $\frac{3.1416cch + 3.1416hhh}{2} \quad \frac{3.1416hhh}{3} = S.$

Which, being reduced, will become $\overline{3\,cch + hhh} \times 0.5236 = S$; which is one Theorem for finding the Solidity of the Fruſtum, and may be expreſſed in Words, thus:

If to three times the Square of the Semidiameter of the Fruſtum's Baſe, you add the Square of the Height of the Fruſtum, and multiply the Sum by the Height of the Fruſtum, and that Product multiplied by .5236, the Product will be the ſolid Content.

But if the Axis of the Sphere, and the Height of the Fruſtum, be given; then put D = the Axis, h = the Height of the Fruſtum, and c as before; it will be $\overline{D - h} \times h = cc$, *viz.* $Dh - hh = cc$. Then will $3\,Dhh - 2hhh = 3cch + hhh$; conſequently, $\overline{3\,Dhh - 2hhh} \times 0.5236 = S$, the Fruſtum's Solidity: Which is another Theorem for finding the Solidity of the Fruſtum, and may be expreſſed in Words, thus:

From three times the Axis ſubtract twice the Height of the Fruſtum, and multiply the Remainder by the Square of the Height, and that Product multiply by .5236, this laſt Product will be the Solidity of the Fruſtum.



Example. Let ABCD be the Fruſtum of a Sphere; ſuppoſe AB (the Diameter of the Fruſtum's Baſe) be 16 Inches, and CD (the Height) 4 Inches; the Solidity is required.

By

By the firſt Rule.

8	
8	
64	Square of the Semidiameter AD.
3	
192	
16	add the Square of CD.
208	
4	multiply by CD.
832	

.5236
832
10472
15708
41888
435.6352

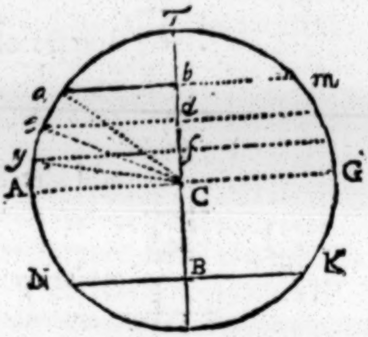
By the ſecond Rule, thus:

Firſt, by the Rule in Page 113, you will find the Axis of the whole Globe to be 20 Inches.

	20 Axis.	.5236	
	3	832	
From	60	10472	
Subtr.	8 twice CD.	15708	
		41888	
Rem.	52		
Mult.	16 Sq. of CD.	435.6352	the ſolid Content, the ſame as before.
	312		
	52		
Prod.	832		

And if it be required to find the middle Part, amNK, uſually called the middle Zone of a Sphere.

Then, becauſe it is ſuppoſed that am = NK, or (which is all one) that bC = CB; therefore it is plain, that if twice the Segment, aTm, be taken from the whole Sphere, there will remain the middle Zone amNK.



But becauſe the Work is a little troubleſome, I will here ſhew how to raiſe a Theorem for the doing it:

Firſt, becauſe AC = yC = eC = aC = TC; therefore it will be □ AC — □ Cf = yf, □ AC — □ CD = □ ed, □ AC — □ Cb = □ ab, &c.

Here becauſe □ AC, □ AC, □ AC, &c. are of Series of Equals, and Cb the Number of all the Terms; therefore □ AC × Cb = the Sum of all that Series (*per* Lemma I.).

And □ Cf, □ Cd, □ Cb, &c. being a Series a Squares, whoſe Roots are in arithmetical Progreſſion, beginning at the Center, C; *viz.* o, Cf, Cd, Cb, &c. wherein the greateſt Term is □ Cb, and the Number of Terms is Cb; therefore □ Cb × $\frac{1}{3}$ Cb = the Sum of all the Series (*per* Lemma III.).

Conſequently, the ⊙ AC × Cb : — ⊙ Cb × $\frac{1}{3}$ Cb = the Sum of all the Series ⊙ yf, ⊙ ed, ⊙ ab, &c. which do conſtitute the Solidity of the half Zone amAG.

And

And becaufe $\square AC - \square Cb = \square ab$; therefore $\odot AC - \odot ab = \odot Cb$. Confequently $\odot AC \times Cb$:

$$\frac{-\odot AC + \odot ab : \times Cb}{3} = 2 \odot AC + \odot ab : \times \tfrac{1}{3} Cb$$

will be the Solidity of the half Zone.

Put $D = AG = 2\,AC$, $x = am$, and $H = bB = 2\,Cb$.

Then $\odot AC = .7854\,DD$, $\odot ab = .7854\,xx$. And if we turn the common Factor .7854 into a Divifor 1.27323, and then take the Triple of that Divifor; *viz.* 3.8197, the Refult of the precedent Work will produce the following Theorem.

$$\text{Theo.} \left\{ \frac{2DD + xx}{3.8197} : \times H = \right\} \text{the middle Zone } amNK.$$

Which in Words is thus: To twice the Square of the Axis AG, and the Square of the Diameter of the Fruftum's Bafe (am), divide the Sum by 3.8197, then multiply the Quotient by the Height or Thicknefs of the middle Zone, and the Product will be the Solidity of the middle Zone required.

This is fo plain and eafy, that it needs no Example.

§ XII. *Of a* SPHEROID.

A Spheroid is a Solid reſembling an Egg. To find the ſolid Content thereof, this is

The RULE.

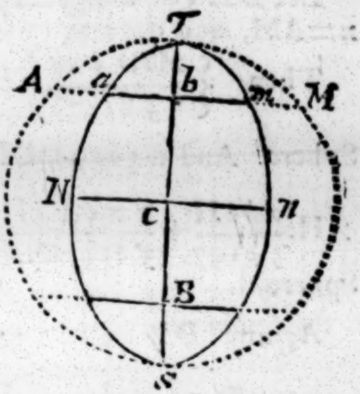
Multiply the Square of the Diameter of the greateſt Circle by the Length, and that Product multiply again by .5236; this laſt Product will be the Solidity of the Spheroid.

Let AB, the Diameter of the greateſt Circle, be 33 Inches, and CD (the Length) 55 Inches, the Solidity is required.

33	59895
33	.5236
99	359370
99	179685
1089	119790
55	299475
5445	31361.0220 the Solidity.
5445	
59895	

Demon-

Demonstration. Every Spheroid is equal to $\frac{2}{3}$ of a Cylinder, whose Base is equal to the greatest Circle of the Spheroid, and its Height equal to the Length of the Spheroid.



Suppose the Figure NTnSN, in the annexed Scheme, to represent a Spheroid, formed by the Rotation of the Semi-Ellipsis TNS, about its transverse Axis TS.

Let D = T S, the Length of the Spheroid, and the Axis of the circumscribing Sphere; and d = Nn, the Diameter of the greatest Circle of the Spheroid:

Then, because □ TC : □ NC : : □ Ab : □ ab, by *Sect.* XV. *Step* 3. *Page* 125.

Therefore it will be, DD : dd : : □ Ab : □ ab.

But the Sum of an infinite Series of such Circles as ⊙ Ab (whose Diameters are Chords) do constitute the Solidity of the Sphere. (By *Sect* XI.)

And the Sum of an infinite Series of such Circles as ⊙ ab (*viz.* whose Diameters are Ordinates of the Ellipsis) do constitute the Solidity of the Spheroid.

Therefore, D D : d d : : 0.5236 D D D : 0.5236 Ddd = the Solidity of the Spheroid. (*Eucl.* 5. 12.)

But 0.5236 Ddd = $\frac{2}{3}$ of the Cylinder, whose Diameter is = d, and Height = D. (By *Sect.* V.)

Now, from this Proportion, between the Sphere and its inscribed Spheroid, it will be very easy to deduce Theorems for finding the solid Content, either of the Frustum or middle Zone of any Spheroid; having the same Height with that of the Sphere; for,

As the Solidity of the whole Sphere is to the Solidity of the whole Spheroid, so is any Part of the Sphere to the like Part of the Spheroid.

As for Inſtance: Suppoſe it was required to find the middle Zone of any Spheroid.

Let $D=TS$, and $d=Nn$, as above; and $H=bB$, $x=AM$, and $c=am$.

Then $\left\{\dfrac{2DD+xx}{3.8197}\right.\times H=$ the middle Zone of the Sphere. And $0.5236\,DDD : 0.5236\,ddD :: \dfrac{2DD+xx}{3.8197}\times H : \dfrac{2ddH}{3.8197}+\dfrac{xxddH}{3.8197DD}=$ the middle Zone of the Spheroid.

Again, $DD : dd :: xx : cc$. Therefore $\dfrac{xxdd}{DD}=cc$. Conſequently, $\dfrac{xxdd}{DD}\times\dfrac{H}{3.8197}=\dfrac{cc}{3.8197}\times H$: Which being taken inſtead of $\dfrac{xxddH}{3.8197DD}$, there will ariſe the following Theorem $\left\{\dfrac{2dd+cc}{3.8197}\right.:\times H=$ the middle Zone of the Spheroid.

Not., That $3.8197=1.2732\times 3$. See *Page* 102.

§ XIII. *Of a Parabolic* CONOID.

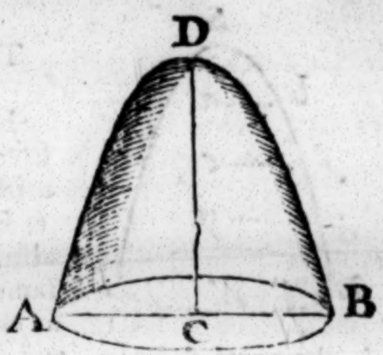
A Parabolic Conoid is ſomething like a half Spheroid, having its Sides ſomewhat ſtraiter. It is generated by ſuppoſing a Semi-parabola turned about its Axis. Te find the ſolid Content thereof, this is

The RULE.

Multiply the Square of the Diameter of its Baſe by .7854, and multiply that Product by half the Height, that laſt Product ſhall be the ſolid Content.

Let

Let ABCD be a Parabolic Conoid, the Diameter of whoſe Baſe is 36 Inches, and its Height CD 33 Inches; the Solidity is required.

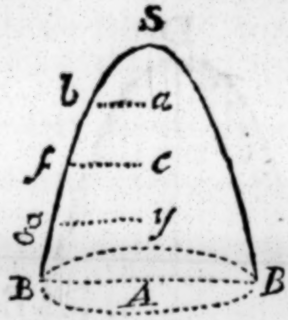


36	.7854	1017.8784
36	1296	33
216	47124	30536352
108	70686	30536352
	15708	
1296	7854	2)33589.9872
	1017.8784	16794.9936

1728)16794.99|36(9.71 Feet, the Content.
15552
———
12249
12096
———
3339
1728
———
1611

Demonſtration. The Parabolic Conoid is conſtituted of an infinite Number of Circles, whoſe Diameters are the Ordinates of the Parabola. Now, according to the Property of every Parabola, it will be, SA : AB : : AB : $\frac{\square AB}{SA}$ = L, the *Latus Rectum.*

Then



$$\text{Then}\begin{cases} Sa \times L = \square\, ba, \\ Se \times L = \square\, fe, \\ Sy \times L = \square\, gy, \ \&c. \end{cases}$$

Here $Sa \times L$, $Se \times L$, $Sy \times L$, &c. are a Series of Terms in arithmet. Progreff. Therefore $\square$ ba, $\square$ fe, $\square$ gy, &c. are alfo a Series of Terms in the fame Progreffion, beginning at the Point S, wherein $\square$ AB is the greateft Term, and SA, the Number of all the Terms. Therefore $\square\, AB \times \frac{1}{2} SA =$ the Sum of all the Series. (By Lemma II.)

Confequently, $\odot\, AB \times \frac{1}{2} SA =$ the Sum of all the Series of $\odot$ ba, $\odot$ fe, $\odot$ gy, &c. which do conftitute the Solidity of the Conoid.

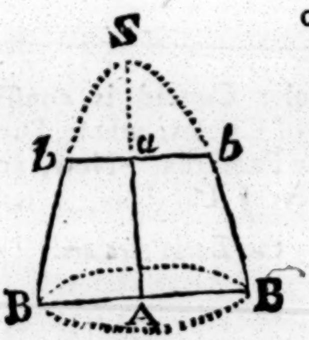
Put $D = 2AB$, and $H = SA$.

Then $.7854\, DD \times \frac{1}{2} H = .3927\, DDH$ will be the folid Content of the Conoid; which is juft half the Cylinder, whofe Bafe is $= D$, and Height $= H$.

This being rightly underftood, it will be eafy to raife a Theorem for finding the lower Fruftum of any Parabolic Conoid.

For, fuppofing $h = aA$, the Height of the Fruftum, and $p = Sa$, the Height of the Part bSb cut off, and $h + p = SA$, the Height of the whole Conoid.

Confequently, $\dfrac{\odot AB \times H + \odot AB \times p}{2} =$ the Solidity of the whole Conoid.



And $\dfrac{\odot ba \times p}{2} =$ the Solidity of the Part cut off.

Therefore

Therefore	1	$\dfrac{\odot AB \times h + \odot AB \times p - \odot ba \times p}{2}$ the Solidity of the Frustum.
But	2	$h+p : \square AB :: p : \square ba$
Conseq	3	$h+p : \bigcirc AB :: p : \bigcirc ba$
$3 \because$	4	$\bigcirc AB \times p := \bigcirc ba \times ph + \odot ba \times p$
$4 - \odot ba \times p$	5	$\odot AB \times p := \odot ba \times h \bigcirc ba \times h$
$\overline{}$ 1×2	6	$\odot AB \times h : + \bigcirc AB \times p : - \odot ba \times p = 2F.$
$6-5$	7	$\odot AB \times h = 2F : - \odot ba \times h.$
$7 + \odot ba \times h$	8	$\odot AB \times h : + \odot ba \times h = 2F.$
— $8 \div 2$	9	$\dfrac{\odot AB + \odot ba}{2} : \times h = F$, the Frustum's Solidity.

Let $D = 2\,AB$, as before, and $d = 2\,ba$, the Diameter of the Part cut off; then we shall have the following Theorem.

$0.3927\,DD + 0.3927\,dd : \times h =$ the Solidity of the Frustum required: Which in Words is thus:

Multiply the Sum of the Squares of the greater and lesser Diameters by .3927, and the Product by the Height of the Frustum, the last Product shall be the solid Content.

§ XIV. *Of a Parabolic* SPINDLE.

IF an acute Parabola be supposed to be moved about its greatest Ordinate, it will form a Solid, called a Parabolic Spindle. To find the solid Content, this is

The RULE.

Multiply the Square of the Diameter of its greatest Circle by .41888 (being $\frac{8}{3}$ of .7854) and that Product by its Length; that last Product is the solid Content.

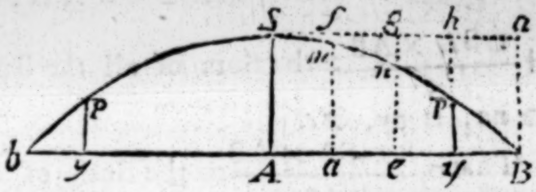
Let ABCD be a Parabolic Spindle, whose greatest Diameter CD is 36 Inches, and its Length AB 99 Inches; the Solidity is required.

```
  36=CD            .41888
  36                 1296
 ----              ------
 216               251328
108               376992
 ----             83776
1296 Square.     41888
                 --------
                 542.86848
                        99
                 ---------
                 488581632
                488581632
                ----------
        1728)53743.97952(31.10184
                 ......
          ------
               1903
                1759
                  3179
                  14515
                    6912
                    ----
                    ....
```

The solid Content is 31.10184 Feet.

Demonſtration. A Parabolic Spindle is conſtituted of an infinite Series of Circles, whoſe Diameters are all parallel to the Axis of the Parabola, as $\odot$ m a, $\odot$ ne, $\odot$ p y, *&c.*

Let us ſuppoſe the Line S d parallel to AB, *&c.* Then it hath already been proved, that the Lines f m, g n, h p, *&c.* are a Series of Squares, whoſe Roots are in arithmetical Progreſſion, conſequently their Squares; *viz.* $\square$ fm, $\square$ g n, $\square$ h p, *&c.* will be a Series of Biquadrats, whoſe Roots will be in arithmetical Progreſſion: Which being premiſed, we may proceed thus:



First	1	SA—fm = m a.
	2	SA—g n = n e.
	3	SA—h p = p y.
1 $\odot$ 2	4	$\square$ SA—2SA $\times$ fm + $\square$ fm = $\square$ m a.
2 $\odot$ 2	5	$\square$ SA—2SA $\times$ gn + $\square$ g n = $\square$ n e.
3 $\odot$ 2	6	$\square$ SA—2SA $\times$ h p + $\square$ h p = $\square$ p y, *&c.*

1. In theſe Equations, the $\square$ S A, $\square$ S A, $\square$ S A, being a Series of Equals, and AB the Number of all the Terms; therefore it will be $\square$ S A $\times$ AB = the Sum of the Series. (By Lemma I.)

2. Becauſe f m, g n, h p, *&c.* are a Series of Squares, wherein S A is the greateſt Term, and A B the Number of all the Terms;

Therefore $\frac{2SA \times SA \times AB}{3} = \frac{2 \square SA \times AB}{3}$ will be the Sum of all the Series. (By Lemma III.)

 3. And

3. And the □ fm, □ gn, □ hp, *&c.* will be a Series of Terms in the Ratio of Biquadrats, as above; □ SA being the greateſt Term, and AB the Number of all the Terms. Therefore it will be $\frac{\square SA \times AB}{5}$ = the Sum of all the Series. (By Lemma V.)

Whence it follows, that $\square SA \times AB - \frac{2\,\square SA \times AB}{3} + \frac{\square SA \times AB}{5}$ = the Sum of all the Series of □ ma, □ ne, □ py, *&c.*

That is, $\frac{8\,\square SA \times AB}{15}$ = the Sum of all the Series □ me, □ ne, □ py, *&c.* Conſequently, $\frac{8 \odot SA \times AB}{15}$ = the Sum of all the Series of Circles, ⊙ ma, ⊙ ne, ⊙ py, *&c.* which conſtitute the Solidity of half the Spindle; *viz.* of SAB.

Therefore putting D = 2SA, and H = 2AB, it will be 0.41888 DDH = the Solidity of the whole Parabolic Spindle bSB, being $\frac{8}{15}$ of 0.7854 DDH, the Solidity of its circumſcribing Cylinder.

From hence we may alſo raiſe a Theorem for finding the Fruſtum, SApy, of the laſt Figure.

For ⊙ SA being the greateſt Term, ⊙ py the leaſt Term, and Ay the Number of all the Terms or Circles included between A and y:

Therefore

Therefore	1	$\square SA - \frac{2SA \times hp}{3} + \frac{\square hp}{5} : \times Ay =$ z the Sum of all the Series, $\square$ SA, $\square$ ma, $\square$ en, $\square$ py.
$\overline{1 \times 3}$	2	$3 \square SA - 2SA \times hp + \frac{3 \square hp}{5} : \times Ay = \overline{3z}$
$z - Ay$	3	$3 \square SA - 2SA \times hp + \frac{3 \square hp}{5} = \frac{3z}{Ay}$
But	4	$\square SA - 2SA \times hp = \square py - \square hp$, *per St.* 6.
$3 - 4$	5	$2 \square SA + \frac{\square 3hp}{5} = \frac{3z}{Ay} - \square py + \square hp.$
$\overline{5} +$ &c.	6	$2 \square SA + \square py - \frac{2}{5} \square hp = \frac{3z}{Ay}$

Conseq. 7 $2 \odot SA + \odot py - \frac{2}{5} \odot hp : \times \frac{1}{3} Ay = z$ the Sum of all the Series of $\odot$SA, $\odot$ma, $\odot$ne, $\odot$py, which do constitute the Solidity of the Frustum S Apy. Therefore putting $D = 2SA$, as before, $C = 2py$. $x = 2hp$, and $H = Ay$; it will be $1.5708DD + .7854CC - .31416xx : \times \frac{1}{3} H =$ the Frustum SApy. And if we make $L = 2H$, then $1.5708DD + .7854CC - .31416xx : \times \frac{1}{3} L =$ the Double of that Frustum, being the middle Zone. Which in Words is thus:

Multiply the Square of the greatest Diameter by 1.5708, and multiply the Square of the lesser Diameter by .7854, and multiply the Square of the Difference of the Diameters by .31416; from the Sum of the two former Products subtract the latter Product, and multiply the Remainder by one-third Part of the Length, and that Product will be the Solidity of the middle Zone required.

CHAP. III.

Of the Measuring the Works of the several Artificers relating to Building; and what Methods and Customs are observed therein.

§ I. *Of* CARPENTERS *Work.*

THE Carpenters Works, which are measurable, are Flooring, Partitioning, and Roofing; all which are measured by the Square of 10 Feet long, and 10 Feet broad; so that one Square contains 100 Square Feet.

1. *Of Flooring.*

If a Floor be 57 Feet 3 Inches long and 28 Feet 6 Inches broad; How many Squares of Flooring are there in that Room?

Multiply 57 Feet 3 Inches by 28 Feet 6 Inches, and the Product is 1631 Feet, *&c.* which divide by 100 (this is done by cutting from the Product two Figures towards the Right Hand with a Dash of the Pen); and the remaining Figures are the Quotient, and

and the Figures cut off are Feet: Thus, 1631 divided by 100, by cutting off 31 from the Right Hand thereof, the Quotient is 16 Squares, and the 31 cut off is 31 Feet.

See the Work both by Decimals, and alfo by Feet and Inches.

57.25		F.	I.	
28.5		57	3	
		28	6	
28625				
45800		456		
11450		114		
		28	7	6
16|31.625		7	0	0
		16|31	7	6

Facit 16 Squares and 31 Feet.

Note, That 5 is the Decimal for half of any thing, .25 is the Decimal for a Quarter, and .125 is the Decimal for half a Quarter; fo in the laft Example, .25 is the Decimal of 3 Inches, becaufe 3 Inches is a Quarter of a Foot; and 5 is the Decimal of 6 Inches, becaufe 6 Inches is half a Foot.

Example 2. Let a Floor be 53 Feet 6 Inches long and 47 Feet 9 Inches broad; How many Squares are contained in that Floor?

47.75		F.	I.	
53.5		53	6	
		47	9	
23875				
14325		371		
23875		212		
		26	9	
25|54.625		13	4	6
		23	6	
		25|54	7	6

Facit 25 Squares and 54 Feet.

By Scale and Compaſſes.

In the firſt Example, extend the Compaſſes from 1 to 28 5, that Extent will reach from 57 25 to 16 Squares and near a third Part.

In the ſecond Example, extend the Compaſſes from 1 to 47.5, that Extent will reach from 53.5 to 25 Squares and above a Half.

2. *Of Partitioning.*

Example 1. If a Partition between Rooms be in Length 82 Feet 6 Inches, and in Height 12 Feet 3 Inches; How many Squares are contained therein?

The Length and Breadth being multiplied together, the Product is 1010.625; which divided by 100 (as before is ſhewed) and the Anſwer is 10 Squares 10 Feet; the Inches or Parts, in theſe Caſes are of no Value.

	F.	I.	
12.25			
82.5	82	6	
———	12	3	
6125	———		
2450	990	0	
9800	20	7	6
———	———		
10|10.625	10|10	7	6

Facit 10 Squares 10 Feet.

Example 2. If a Partition between Rooms be in Length 91 Feet 9 Inches, and its Breadth 11 Feet 3 Inches; How many Squares are contained therein?

The Length and Breadth being multiplied together, the Product is 1032 Feet; which divided by 100, the Anſwer will be 10 Squares and 32 Feet.

91.75

91.75		F.	I.	
11.25		61	9	
		11	3	
45875				
18350		1009	3	
9175		22	11	3
9175				
10|32.1875		10|32	2	3

3. *Of Roofing.*

It is a Rule amongst Workmen, that the Flat of any House, and half the Flat thereof, taken within the Walls, is equal to the Measure of the Roof of the same House; but this is when the Roof is true pitched: For if the Roof be more flat or steep than the true Pitch, it will measure to more or less accordingly.

Example 1. If a House within the Walls be 44 Feet 6 Inches long, and 18 Feet 3 Inches broad; How many Squares of Roofing will cover that House?

Multiply the Length and Breadth together, and the Product is 812 Feet, the Flat; the half thereof is 406 Feet; which added to the Flat, the Sum is 1218 Feet; which divided by 100, the Answer is 12 Squares and 18 Feet.

	18.25		F.	I.	
	44.5		44	6	
			18	3	
	9125				
	7300		352		
	7300		44		
			11	1	6
Flat	812.125		9	0	0
Half	406				
		The Flat	812	1	6
	12|18	The Half	406		
		Sum	12|18		

Facit 12 Squares 18 Feet.

By Scale and Compaffes.

In the firft Example of Partitioning, extend the Compaffes from 1 to 12.25, that Extent will reach from 82.5 to 10 Squares and One Tenth.

In the fecond Example, extend the Compaffes from 1 to 11.25. that Extent will reach from 91.75 to 10 Squares, and a little lefs than a third Part.

In the Example of Roofing extend the Compaffes from 1 to 18.25, that Extent will reach from 44 5 to 812, the Flat; to which add the half thereof, and the Sum is 12.18; which is 12 Squares 18 Feet, as above.

There are other Works about a Building, done by the Carpenter, which are meafured by the Foot, running Meafure, that is, by the Number of Feet in Length only; as Cornices, Doors and Cafes, Window-frames, Guttering, Lintels, Sommers, Skirtboards, *&c.*

Note 1. In the Meafuring of Flooring, after you have meafured the whole Floor, you muft deduct out of it the Well-holes for the Stairs and Chimnies; and in Partitioning, for the Doors, Windows, *&c.* except (by Agreement) they are to be included.

Note

Note 2. In measuring of Roofing, seldom any Reductions are made for the Holes for the Chimney-shafts, the Vacancies for Lutheren-lights and Sky-lights; for they are more Trouble to the Workman than the Stuff which would cover them is worth.

§ II. *Of* BRICKLAYERS *Work*.

THE principal is Tiling, Walling, and Chimney-work.

1. *Of Tiling*.

Tiling is measured by the Square of 100 Feet, as Flooring, Partitioning, and Roofing were in the Carpenters Work; so that between the Roofing and Tileing, the Difference will not be much; yet the Tiling will be the most; for the Bricklayers sometimes will require to have double Measure for Hips and Vallies. When Gutters are allowed double Measure, the way is to measure the Length along the Ridge-tile, and by that means the Measure of the Gutters becomes double; it is usual also to allow double Measure at the Eaves, so much as the Projector is over the Plate, which is commonly about 18 or 20 Inches.

Example 1. There is a Roof covered with Tiles, whose Depth on both Sides (with the usual Allowance at the Eaves) is 37 Feet 3 Inches, and the Length 45 Feet; I demand how many Squares of Tiling are contained therein?

F.	I.			37.25
37	3			45
45	0			18625
185				14900
148				16|762.5
11	3			
16|76	3			

Anſwer, 16 Squares 76 Feet.

Example 2. There is a Roof covered with Tiles, whoſe Depth on both Sides (with the Allowance at the Eaves) is 35 Feet 9 Inches, and the Length 43 Feet 6 Inches; I demand how many Squares of Tileing are in the Roof?

F.	I.			35.75
43	6			43.5
35	9			17875
215				10725
129				14300
21	9			15|55.125
10	10	6		
17	6			
15|55	1	6		

Here the Length and Depth being multiplied together, the Product is 1555 Feet; which divided by 100 (as before is taught) the Anſwer is 15 Squares and 55 Feet.

By Scale and Compaſſes.

In the firſt Example, extend the Compaſſes from 1 to 37.25, that Extent will reach from 45 to 16 Squares and a little above three Quarters of a Square.

In

In the ſecond Example, extend the Compaſſes from 1 to 35.75, that Extent will reach from 43.5 to 15 Squares and 35 Feet; that is, a little above a Half-ſquare.

2. *Of Walling.*

Bricklayers commonly meaſure their Work by the Rod Square of 16 Feet and a half; ſo that one Rod in Length, and one in Breadth, contain 272.25 Square Feet; for 16.5, multiplied in itſelf, produces 272.25 Square Feet. But in ſome Places the Cuſtom is to allow 18 Feet to the Rod; that is, 324 Square Feet. And in ſome Places the uſual Way is, to meaſure by the Rod of 21 Feet long and 3 Feet high, that is, 63 Square Feet; and here they never regard the Thickneſs of the Wall, but the uſual Way is to moderate the Price according to the Thickneſs.

When you meaſure a Piece of Brick-work, the firſt thing is to inquire by which of thoſe Ways it muſt be meaſured; then, having multiplied the Length and Breadth in Feet together, divide the Product by the proper Diviſor, either for Rods or Roods, and the Quotient is Square Rods, or Square Roods, accordingly.

But commonly Brick-walls, that are meaſured by the Rod, are to be reduced to a Standard-thickneſs; *viz.* of a Brick and a half thick (if it be not agreed on the contrary); and to reduce a Wall to Standard-thickneſs, this is

The RULE.

Multiply the Number of ſuperficial Feet that are found to be contained in any Wall by the Number of Half-bricks which that Wall is in Thickneſs; one third Part of that Product ſhall be the Content thereof in Feet, reduced to the Standard-thickneſs of one Brick and a half.

Example

Example 1. If a Wall be 72 Feet 6 Inches long, and 19 Feet 3 Inches high, and 5 Bricks and a half thick; How many Rods of Brick-work are contained therein, when reduced to the Standard?

```
          19.25  Height.
          72.5   Length
         ------
           9625
          3850
         13475
        -------
        1395.625
              11
      ----------
     3)15351.875
      ----------
272.25) 5117.291(18 Rods.
        --------
          239479
        --------
    68.06)21679(3 Quarters of a Rod.
          -----
          12.61
```

Anſwer, 18 Rods 3 Quarters 12 Feet.

F.

F.	I.	
72	6	
19	3	
648		
72		
18	1	6
9	6	0
		11

3)12345

272)5115(18 Rods.

2395

68)219(3 Quarters of a Rod.

15

Note, That 68.06 is one-fourth Part of 272.25.

Note also, That in reducing of Feet into Rods, they usually reject the odd Parts, and divide only by 272, as is done in the second Way of the last Example; so the Answer, by the second Way, is 18 Rods 3 Quarters and 15 Feet; more by about $2\frac{1}{2}$ Feet than by the first Way, where it is done decimally; a thing very insignificant.

Example

Example 2. If a Wall be 245 Feet 9 Inches long, 16 Feet 6 Inches high, and two Bricks and a half thick; I demand how many Rods of Brick-work are contained therein, when reduced to Standard-thickneſs?

```
       245.75
         16.5
      -------
       122875
      147450
      24575
     --------
     4054.875
           5
    ---------
  3)20270
   --------
272)6756(24 Rods.
    -----
     1316
     ----
   68)228(3 Quarters of a Rod.
      ---
       24
```

Anſwer, 24 Rods 3 Quarters 24 Feet.

```
     F.  I.
    245  9
     16  6
   --------
   1470
   245
    122 10  6
     12  0  0
   -----------|
   4054 10  6 Anſwer in Feet.
```

Before I shew how to work the two last Examples by Scale and Compasses, I will shew how to find proper Divisors to facilitate the Operation; because it would be too intricate and tedious to perform by Scale and Compasses, according to the Rule above taught.

To find proper Divisors.

Divide 3 (the Number of Half bricks in 1 $\frac{1}{2}$), by the Number of Half-bricks in the Thickness, the Quotient will be a Divisor, which will give the Answer in Feet. But if you would have a Divisor to bring the Answer in Rods at once, then multiply 272.25 by the Divisor found for Feet, and the Product will be a Divisor, which will give the Answer in Rods.

Example. Let it be required to find a Divisor proper to reduce a Wall of three Bricks thick.

Divide 3 by 6 (the half-bricks in the Thickness) and the Quotient is .5, which is a Divisor that will give the Answer in Feet. Then multiply 272.25 by .5, and the Product is 136.125, the Divisor, which will give the Answer in Rods; that is, as 136.125 is to the Length of the Wall, so is the Height to the Content in Rods. Or, as .5 is to the Length, so is the Height to the Content in Feet.

After the same manner you may find Divisors for any other Thickness, which you will find to be as expressed in the following little Table.

The Thicknefs of the Wall.	Divifors for the Anfwer in Feet.	Divifors for bringing the Anfwer in Rods.
1 Brick thick	1.5	408.375
$1\frac{1}{2}$ Brick thick	1.	272.25
2 Bricks thick	.75	204.1875
$2\frac{1}{2}$ Bricks thick	.6	163.35
3 Bricks thick	.5	136.125
$3\frac{1}{2}$ Bricks thick	.4285	116.659
4 Bricks thick	.375	102.0937

Let the fecond Example, aforegoing, be wrought by Scale and Compaffes, where the Length is 245.75, the Height 16.5, and the Thicknefs $2\frac{1}{2}$ Bricks.

Extend the Compaffes from 163.35 (the tabular Number againft $2\frac{1}{2}$ Bricks), to 245.75; that Extent will reach from 16.5 to 24 Rods and 8 Tenths.

Again, if the Length be 75 Feet 6 Inches, and the Height 18 Feet 9 Inches, at $3\frac{1}{2}$ Bricks thick; How many Rods are contained therein?

Extend the Compaffes from 116.659 (the tabular Number) to 18.75, that Extent will reach from 75.5 to 12.13, that is, 12 Rods and a little above half a Quarter.

It will be very proper and commodious, for fuch as have frequent Occafion to meafure Brick-work, to have in the Line of Numbers little Brafs Center-pins at each of the Numbers in the third Column of the above little Table, with a Figure to denote the Thicknefs of the Wall.

If

If a Wall be 104 Feet 9 Inches long, and 17 Feet 3 Inches high; How many Rods are contained therein?

104.75	F.	I.	
17.25	104	2	
	17	3	
52375	728		
20950	104		
73325	26	2	3
10475	12	9	0
63)1806.9375(28			
126	1806	11	3

Answer, 28 Rods 42 Feet.

546
504

42

Note, That such as dig Cellars, do many times make them by the Floor, 16 Feet square, and a Foot deep, being a Floor of Earth; that is, 324 solid Feet.

3. *Of Chimnies.*

If you are to measure a Chimney standing alone by itself, without any Party-wall being adjoined, then girt it about for the Length, and the Height of the Story is the Breadth; the Thickness must be the same as the Jambs are of, provided that the Chimney be wrought upright from the Mantle-tree to the Cieling, not deducting any thing for the Vacancy between the Floor (or Hearth) and the Mantle-tree, because of the Gatherings of the Breast and Wings, to make room for the Hearth in the next Story.

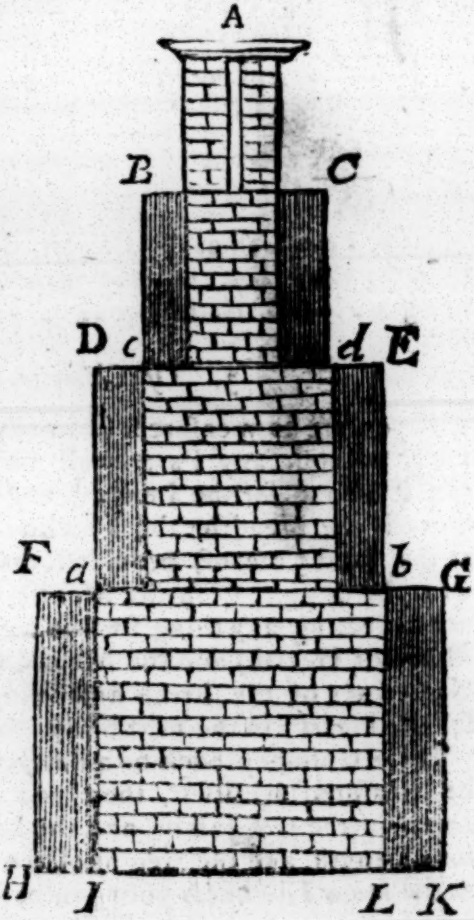
If the Chimney-back be a Party-wall, and the Wall be measured by itself, then you must measure the Depth of the two Jambs, and the Length of the Breast for a Length, and the Height of the Story the Breadth, at the same Thickness your Jambs were of.

When you measure Chimney-shafts, girt them with a Line round about the least Place of them, for the

the Length, and the Height ſhall be your Breadth: And if they be four Inch-work, then you muſt ſet down their Thickneſs at one Brick-work; but if they be wrought 9 Inches thick (as ſometimes they are, when they ſtand high and alone above the Roof), then you muſt account your Thickneſs $1\frac{1}{2}$ Brick, in conſideration of Withs and Pargetting, and Trouble in Scaffolding.

It is cuſtomary, in moſt Places, to allow double Meaſure for Chimnies.

Example. Suppoſe this Figure, ABCDEFGHIK, to be a Chimney that hath a double Tunnel towards the Top, and a double Shaft, and is to be meaſured according to double Meaſure.



Firſt, I begin with the Breaſt-wall IL, and the two Angles LK and HI, which together are 18 Feet 9 Inches; then take the Height of the Square HF, 12 Feet 6 Inches; which multiplied together, produce 234 Feet 4 Inches 6 Parts, for the Content of the Figure FGHK.

For

For the Square DaEb, the Length of the Breast-wall and two Angles, is 14 Feet 6 Inches, and the Height Da 9 Feet; which multiplied together, make 130 Feet 6 Inches, for the Content of the Part DaEb.

Then the Height of the next Square 7 Feet, and the Length of the Breast wall and two Angles is 10 Feet 3 Inches; which multiplied together, produceth 71 Feet 9 Inches, for the Content of the Square BcCd.

The Compass of the Chimney-shafts is 13 Feet 9 Inches, and the Height 6 Feet 6 Inches; which multiplied together, make 89 Feet 4 Inches 6 Parts, the Content of the Shafts.

The Depth of the middle Fetter, that parts the Funnels, is 12 Feet, and its Wideness 1 Foot 3 Inches; which multiplied together, make 15 Feet, the Content thereof.

The Work.

	F.	I.			
	18	9			18.75
	12	6			12.5
					9375
	225	0			3750
	9	4	6		1875
FGHK	234	4	6	FGHK	234.375

	F.	I.		
	14	6		14.5
	9	0		9
DaEb	130	6	DaEb	130.5

	F.	I.		
	10	3		10.25
	7	0		7
BcCd	71	9	BcCd	71.75

	F.	I.	
	13	9	
	6	6	
	82	6	
	6	10	6
The Shaft	89	4	6

	13.73
	6.5
	6875
	8250
The Shaft	89.375

	F.	I.
	1	3
	12	0
The Fetter	15	0

	1.25
	12
The Fetter	15.00

272)1082(3 Rods.

68) 266(3 Quarters.

Rem. 62 Feet.

	F.	I.	P.
FGHK	234	4	6
DaEb	130	6	0
BcCd	71	9	0
The Shaft	89	4	6
The Fetter	15	0	0
The Sum	541	0	0
The Double	1082	0	0

Having added the five Products together, and doubled the Sum, that double Sum is the Content of the Chimney in Feet, according to double or cuſtomary Meaſure; which Feet muſt be reduced to Rods, as was ſhewed before.

So the Feet in the foregoing Example being reduced to Rods (the Thickneſs being ſuppoſed 1 ½ Bricks) it makes 3 Rods 3 Quarters and 62 Feet; that is, 4 Rods wanting 6 Feet.

This is all the Meaſure that can be allowed, when the Chimney ſtands in a Gavel or Side-wall; in which Caſe the Back of the Chimney (here not meaſured) is accounted as Part of the Gavel; but if the Chimnies ſtand by themſelves, as all Stacks of Chimnies in

in great Buildings do, in such Case, it is all Chimney-work, and therefore ought to be measured double on all Sides.

§ III. *Of* PLASTERERS *Work*.

THE Plasterers Works are principally of two Kinds; namely, 1. Works lathed or plastered, which they call Cieling. 2. Works rendered; which are of two Kinds; *viz.* upon Brick-walls, or between Quarters, in the Partitions between Rooms: all which are measured by the Yard-square, or Square, of 3 Feet, which is 9 Feet.

1. *Of Cieling.*

If a Cieling be 59 Feet 9 Inches long, and 24 Feet 6 Inches broad; How many Yards doth that Cieling contain?

Multiply 59 Feet 9 Inches by 24 Feet 6 Inches, and the Product is 1463 Feet 10 Inches 6 Parts; which divided by 9, the Quotient is 162 Yards 5 Feet.

F.	I.	
59	9	
24	6	
236		
118		
29	10	6
18	0	0
1463	10	6

```
        59.75
         24.5
      -------
        29875
       23900
      11950
   ----------
   9)1463.875
   ----------
Answer 162.65
```

By Scale and Compaſſes.

Extend the Compaſſes from 9 to 59 Feet 9 Inches, that Extent will reach from 24 Feet 6 Inches to 16.25 Yards.

2. *Of Rendering.*

Example. If the Partitions between Rooms be 141 Feet 6 Inches about, and 11 Feet 3 Inches high; How many Yards are in thoſe Partitions?

Multiply 141 Feet 6 Inches by 11 Feet 3 Inches, and the Product is 1591 Feet 10 Inches 6 Parts; which divided by 9, gives 176 Yards 7 Feet, the Anſwer.

	F.	I.			
	141	6			141.5
	11	3			11.25
	1556	6			7075
	35	4	6		2830
					1415
	9)1591	10	6		1415
Anſwer	176	7			9)1591.875
					176.87

Anſwer, 176.87 Yards.

Extend the Compaſſes from 9 to 141.5, that Extent will reach from 11.25 to 176.87 Yards.

Note 1. If there be any Doors, Windows, or the like, in your Partitioning, you muſt make Deductions for them.

Note 2. When you meaſure Rendering upon Brick-walls, you are to make no Deductions; but when you meaſure Rendering between Quarters, you may very well deduct one-fifth Part for the Quarters, Braces, and Entertices.

Note

Note 3. That Whiting and Colouring are both meaſured by the Yard, as Cieling and Rendering were; and as in Rendering between Quarters, you deduct one-fifth Part, ſo in Whiting and Colouring you muſt add one-fourth or one-fifth Part at leaſt.

§ IV. *Of* JOYNERS *Work*.

JOyners meaſure their Work by the Yard-ſquare; but in taking their Dimenſions, they differ from ſome others; for they have a Cuſtom, and ſay, *We ought to meaſure where our Plane touches*: Wherefore, in taking the Height of any Room, where there is a Cornice about, and ſwelling Panels and Mouldings, they, with a String, begin at the Top, and girt over all the Mouldings; which will make the Room to meaſure much higher than it is: Then for meaſuring about the Room, they only take it as it is upon the Floor.

Example 1. If a Room of Wainſcot (being girt downwards over the Mouldings) be 15 Feet 9 Inches high, add 126 Feet 3 Inches in Compaſs; How many Yards doth that Room contain?

Multiply the Compaſs by the Height, and the Product is 1988 Feet 5 Inches 3 Parts; which divided by 9, gives 220 Yards and 8 Feet, the Anſwer.

F.	I.		
126	3		126.25
15	9		15.75
620			63125
126			88375
63	1	6	63125
31	6	9	12625
3	9	0	
			9)1988.4375
9)1988	5	3	
			220.8
Anſwer, 220	8		

Facit 220 Yards 8 Feet.

Example 2. If a Room of Wainſcot be 16 Feet 3 Inches high (being girt over the Mouldings), and the Compaſs of the Room 137 Feet 6 Inches; How many Yards are contained therein?

Multiply 137 Feet 6 Inches by 16 Feet 3 Inches, and the Product is 2234 Feet 4 Inches 6 Parts; which divided by 9, the Quotient is 248 Yards and 2 Feet.

F.	I.		
137	6		137.5
16	3		16.25
830			6875
137			2750
34	4	6	8250
			1375
9)2234	4	6	
			9)2234.375
248	2	0	
			248 2

Facit 248 Yards 2 Feet.

By

By Scale and Compaſſes.

For the firſt Example, extend the Compaſſes from 9 to 126.25, that Extent will reach from 15.75 to 220.9 Yards.

For the ſecond Example, extend the Compaſſes from 9 to 137 5, that Extent will reach from 16.25 to 248 Yards and above a Quarter.

In Joyners Work there is another thing to be obſerved; that is, in the meaſuring of Doors, Window-ſhutters, and all ſuch Work as is wrought on both Sides, they are paid for Work and Half work; ſo that in meaſuring all ſuch Work, you muſt firſt find the Content, as before, and take half that Content, and add to it; ſo ſhall the Sum be the Content at Work and half.

Example. If the Window-ſhutters about a Room be 69 Feet 9 Inches broad, and 6 Feet 3 Inches high; How many Yards are contained therein at Work and half?

Multiply 69 Feet 9 Inches by 6 Feet 3 Inches, and the Product is 435 Feet 11 Inches 3 Parts; the Half whereof is 217 Feet 11 Inches 7 Parts; which added together, the Sum is 653 Feet 10 Inches 10 Parts; which divided by 9, the Quotient is 72 Yards 5 Feet, the Content at Work and half.

F.	I.		
69	9		69.75
6	3		6.25
418	6		34875
17	5	3	13950
			41850
435	11	3	435.9375
217	11	7	217.9687
9)653	10	10	653.9062
72	5		

Facit 248 Yards 5 Feet.

By

By Scale and Compaſſes.

Extend the Compaſſes from 9 to 69.75, that Extent will reach from 6.25 to 48.4 Yards; the Half whereof is 24.02; which added together, make 72.6 Yards, the Content at Work and half.

Note, That you muſt make Deductions for all Window-lights; but you muſt meaſure the Window-boards, Sopheta-boards, and Cheeks, by themſelves.

§ V. *Of* PAINTERS *Work.*

THE taking the Dimenſions of Painters Work is the ſame as that of Joyners, by girting over the Mouldings and ſwelling Panels, in taking the Height; and it is but Reaſon that they ſhould be paid for that on which their Time and Colour are both expended. The Dimenſions thus taken, the caſting up, and reducing Feet into Yards, is altogether the ſame as the Joyners Work; but the Painter never requires Work and half, but reckons his Work once, twice, or thrice coloured over. Only take Notice, that Window-lights, Window-bars, Caſements, and ſuch-like Things, they do at ſo much *per* Piece.

Example. If a Room be painted, whoſe Height (being girt over the Mouldings) is 16 Feet 6 Inches, and the Compaſs of the Room 97 Feet 9 Inches; How many Yards are in that Room?

Multiply 97 Feet 9 Inches by 16 Feet 6 Inches, and the Product is 1612 Feet 0 Inches 6 Parts; which being divided by 9, the Quotient is 179 Yards and 1 Foot.

F.	I.		
97	9		97.75
16	6		16.5
584			48875
98			58650
48	10	6	9775
9)1612	10	6	9)1612.875(179
179	1		

Facit 179 Yards 1 Foot.

By Scale and Compasses.

Extend the Compasses from 9 to 16.5, that Extent will reach from 97.75 to 179.2 Yards.

§ IV. *Of* GLASIERS *Work.*

GLasiers measure their Work by the Foot square; so that the Length and Breadth of a Pane of Glass in Feet, being multiplied into each other, produceth the Content.

Note, That Glasiers usually take their Dimensions to a Quarter of an Inch; and in multiplying Feet, Inches, and Parts, the Inch is divided into 12 Parts, as the Foot is, and each Part subdivided into 12, &c.

Example. If a Pane of Glass be 4 Feet 8 Inches and 3 Quarters long, and 1 Foot 4 Inches 1 Quarter broad; How many Feet of Glass are in that Pane?

$$\text{The Decimal of} \left\{ \begin{array}{l} 8 \text{ Inches } \frac{3}{4} \\ 4 \text{ Inches } \frac{1}{4} \end{array} \right\} \text{ is } \left\{ \begin{array}{l} .728 \\ .354 \end{array} \right.$$

F.	I.	P.			
4	8	9			4.729
1	4	3			1.354
					18916
4	8	9			23645
1	6	11	0		14187
	1	2	2	3	4729
6	4	10	2	3	6.403066

Anfwer, 6 Feet 4 Inches.

By Scale and Compaffes.

Extend the Compaffes from 1 to 1.354, that Extent will reach from 4.729 to 6.4 Feet, the Content.

Example 2. If there be 8 Panes of Glafs, each 4 Feet 7 Inches 3 Quarters long, and 1 Foot 5 Inches 1 Quarter broad; How many Feet of Glafs are contained in the faid 8 Panes?

The Decimal of $\left\{\begin{matrix} 7 \text{ Inches } \frac{3}{4} \\ 5 \text{ Inches } \frac{1}{4} \end{matrix}\right\}$ is $\left\{\begin{matrix} .646 \\ .437 \end{matrix}\right.$

F.	I.	P.			
4	7	9			4.646
1	5	3			1.437
					32522
4	7	9			13938
1	11	2	9		18584
	1	1	11	3	4646
6	8	1	8	3	6.676302
				8	8
53	5	1	6	0	53.410416

Facit 53 Feet 5 Inches.

By

By Scale and Compasses.

Extend the Compasses from 1 to 1.437, that Extent will reach from 4.646 to 6.676; then extend the Compasses from 1 to 8, that Extent will reach from 6.676 to 53 4, the Content.

Example 3. If there be 16 Panes of Glass, each 4 Feet 5 Inches and a half long, and 1 Foot 4 Inches 3 Quarters broad; How many Feet of Glass are contained therein?

F.	I.	P.				
F.	I.	P.				4.458
4	5	6				1.395
1	4	9				
						22290
4	5	6				40122
1	5	10	0			13374
	3	4	1	6		4458
6	2	8	1	6		6.218910
				4		4
24	10	8	6	0		24.875640
				4		4
96	6	10	0	0		99.502560

Facit 99 Feet 6 Inches.

Note, That instead of multiplying by 16, I have multiplied by 4 twice, because 4 times 4 is 16.

By Scale and Compasses.

Extend the Compasses from 1 to 1.395, that Extent will reach from 4 458 to 6.219; then extend the Compasses from 1 to 16, that Extent will reach from 6.219 to 99 5 Feet, the Content.

Note, That when Windows have Half-rounds at the Top, they measure them at the full Height, as if they were square. Also round or oval Windows are mea-

ſured at the full Length and Breadth of their Diameters. Likewiſe Crotchet-windows in Stone-work are all meaſured by their full Squares. And there is Reaſon for ſo doing; for the Trouble in taking their Dimenſions to work by, the Waſte of Glaſs in working, and the Time expended in ſetting up, is far more than the Glaſs can be valued at.

§ V. *Of* Masons *Work.*

MAſons meaſure their Work ſometimes by the Foot ſolid, ſometimes by the Foot ſuperficial; and in ſome Places they meaſure their Walling by the Rood, that is, 21 Feet long and 3 Feet high, which is 63 ſquare Feet. Examples of each are as follow.

Example. If a Wall be 97 Feet 5 Inches long, 18 Feet 3 Inches high, and 2 Feet 3 Inches thick: How many ſolid Feet are contained in that Wall?

F.	I.			
97	5			97.417
18	3			18.25
776				487085
97				194834
24	4	3		779336
6	0	0		97417
1	6	0		
				1777.86025
1777	10	3		2.25
2	3			
				888930125
3555	8	6		355572050
444	5	6	9	355572050
4000	2	0	9	4000.1855625

Multi-

Multiply the Length, Height, and Thickneſs together, and the laſt Product is 4000 Feet 2 Inches, the ſolid Feet contained in the Wall.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 18.25, that Extent will reach from 97.417 to 1777.86; then extend from 1 to 1777.86, that Extent will reach from 2.25 to 4000.18, the ſolid Content.

Example 2. If a Wall be 107 Feet 9 Inches long and 20 Feet 6 Inches high; How many Feet ſuperficial are contained therein?

F.	I.			
107	9			107.75
20	6			20.5
2155	0			53875
53	10	6		215500
2208	10	6		2208 875

Facit 2208 Feet 10 Inches.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 107.75, that Extent will reach from 20 6 to 2208.875, the ſuperficial Feet.

Example 3. If a Wall be 112 Feet 3 Inches long and 16 Feet 6 Inches high; How many Roods are contained therein?

F.	I.	
112	3	
16	6	
676	0	
112		
56	1	6
1852	1	6

```
   112.25
     16 5
   ------
    56125
   67350
  11225
  --------
63)1852.125(29
   --------
    592
   ----
     25
```

Facit 29 Roods 25 Feet.

By Scale and Compasses.

Extend the Compasses from 63 to 16.5, that Extent will reach from 112.25 to 29.4 Roods, the Content.

CHAP.

CHAP. IV.

The Measuring of BOARD *and* TIMBER.

§ I. *Of* BOARD MEASURE.

TO measure a Board, is no other but to measure a long Square.

Example 1. If a Board be 16 Inches broad and 13 Feet long; How many Feet are contained therein?

Multiply 16 by 13, and the Product is 208; which divided by 12, gives 17 Feet, and 4 remains, which is a third Part of a Foot.

Or thus: Multiply 156 (the Length in Inches) by 16, and the Product is 2496; which divided by 144, the Quotient is 17 Feet, and 48 remains, which is a third Part of 144, the same as before.

$$\begin{array}{r} 12 : 13 :: 16 \\ 13 \\ \hline 48 \\ 16 \\ \hline 12)208 \\ \hline 17\frac{4}{12} \end{array}$$

$$\begin{array}{r} \text{Or, } 144 : 156 :: 16 \\ 16 \\ \hline 936 \\ 156 \\ \hline 144)2496(17\frac{48}{144} \\ 1056 \\ \hline 48 \end{array}$$

By Scale and Compasses.

Extend the Compasses from 12 to 13, that Extent will reach from 16 to $17\frac{1}{3}$ Feet, the Content.

Or, extend from 144 to 156 (the Length in Inches) that Extent will reach from 16 to $17\frac{1}{3}$ Feet, the Content.

Example 2. If a Board be 9 Inches broad; How many Inches in Length will make a Foot?

Divide 144 by 19, and the Quotient is 7.58 very near; and so many Inches in Length, if a Board be 19 Inches broad, will make a Foot.

Inch.	Inch.	Inch.	Inch.
19 :	144 ::	1 :	7.58 *ferè*.

Extend

Extend the Compasses from 19 to 144, that Extent will reach from 1 to 7.58; that is, 7 Inches, and something more than a half. So, if a Board be 19 Inches broad, if you take 7 Inches, and a little more than a half with your Compasses from a Scale of Inches, and run that Extent along the Board, from End to End, you may find how many Feet that Board contains; or you may cut off from that Board any Number of Feet desired.

For this Purpose there is a Line upon most ordinary Joint-rules, with a little Table placed upon the End of all such Numbers as exceed the Length of the Rule, as in this little Table annexed.

0	0	0	0	5	0	$8\frac{1}{2}$	6
12	6	4	3	2	2	1	1
1	2	3	4	5	6	7	8

Here you see, if the Breadth be one Inch, the Length must be 12 Feet; if two Inches, the Length is 6 Feet; if five Inches broad, the Length is 2 Feet 5 Inches, &c.

The rest of the Lengths are expressed in the Line, thus: If the Breadth be 9 Inches, you will find it against 15 Inches, counted from the other End of the Rule; if the Breadth be 11 Inches, then a little above 13 Inches will be the Length of a Foot, &c.

§ II. *Of* Squared Timber.

By Squared Timber are here meant such as have equal Bases, and the Sides strait and parallel. The Rules for measuring all such Solids are shewed in § II. of Chap. II. to which I refer.

Example

Example 1. If a Piece of Timber be 1 Foot 3 Inches (or 15 Inches) square and 18 Feet long; How many solid Feet are contained therein?

```
        15
        15
        --
        75
       15
       ---
       225
        18
       ----
      1800
      225
      ----
144)4050(28.125
      ----
      1170
      ----
      180
       360
        720
```

```
F.  I.
1   3
1   3
------
1   3
    3   9
---------
1   6   9
        6
---------
9   4   6
        3
---------
28  1   6
```

Answer, 28 Feet and half a Quarter.

Here, instead of multiplying by 18, where I wrought by Feet and Inches, I multiplied by 6, and then by 3, because 3 times 6 is 18.

Example 2. If a Piece of squared Timber be 2 Feet 9 Inches deep, 1 Foot 7 Inches broad, and 16 Feet 9 Inches long; How many Feet of Timber are in that Piece?

Multiply the Depth, Breadth, and Length together, and the Product will be the Content.

33
19

297
33

627
16.75

3135
4389
3762
627

144)10502.25(72.93

422
1342
465

93

F. I.
2 9
1 7

2 9
1 7 3

4 4 3
16 8 0

69 9 0
3 3 2 3

72 11 2 3

Anſwer, 72 Feet 11 Inches; or 72 Feet 93 Parts.

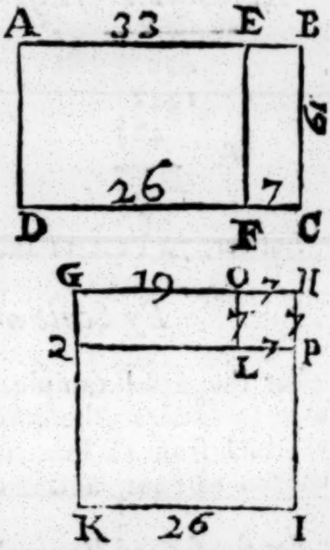
By Scale and Compaſſes.

For the firſt Example, extend the Compaſſes from 12 to 15 Inches (the Side of the Square) that Extent will reach from 18 Feet (the Length being twice turned over) to 28 Feet, and ſomething more.

For the ſecond Example, find a mean Proportional between 19 Inches and 33 Inches, by dividing the Space between them into two equal Parts, and the Compaſs Point will reſt upon 25, which is a mean Proportional between 19 and 33.

Then extend the Compaſſes from 12 to 25 (the Proportional found) that Extent will reach (being twice turned over) from 16.75 Feet, the Length, to 7293 Feet, the Content.

A common Error is committed, for want of Art, in meaſuring theſe laſt Sorts of Solids, by adding the Depth and Breadth together, and taking half for the Side of a mean Square. This Error, though it be but ſmall, when the Depth and Breadth are pretty near equal; yet if the Difference be great, the Error is very conſiderable; for the Piece of Timber thus meaſured, will be more than the Truth, by a Piece whoſe Length is equal to the Length of the Piece of Timber to be meaſured, and the Square equal to half the Difference of the Breadth and Depth, as I ſhall here demonſtrate.



I ſay, the Square GHIK is greater than the Parallelogram ABCD, by the little Square OHPL; for the Parallelogram QPIK is equal to the Parallelogram AEFD; and the Parallelogram GOLQ is equal to the Parallelogram EBCF. Therefore the Square is greater than the Parallelogram by the little Square OHPL; which was to be proved.

Otherwiſe, you may prove it by Numbers, thus: The Sum of 33 and 19 is 52, the half thereof is 26, the Square of 26 is 676, and the Product of the Depth and Breadth is 627: The Difference of theſe two is 49, equal to the Square of half the Difference; for the Difference between 33 and 19 is 14, the half thereof is 7, whoſe Square is 49; which was to be proved.

Now, if this 49 be multiplied by the Length of the Piece, and that Product divided by 144, to bring it to Feet, and thoſe Feet added to the true Content, the Sum

Sum will be equal to the Content found by the false Way mentioned.

See the Work of both.

```
     53 Depth.           16.75 the Length.
     19 Breadth.            49 the Sq. of ½ Diff.
    ---                  -----
     52 Sum.             15075
    ---                  6700
     26 half.            -----
     26              144)820.75(5.69
    ---                  -----
    156                  1007
     52                   1435
    ---                  -----
    676                    139
  16.75
  -----
   3380
  4732
 4056
 676
 ------
144)11323.00(78.63
 ------
 1243
   910
    460
    ---
     28
```

	Feet.	
To	72.93	the true Content.
Add	5.69	the Part superfluous.
Rem.	78.62	equal to the Content by the false Way.

By Feet and Inches.

F. I.
0 7
0 7

0 4 1
16 9

5 5 4
3 0 9

5 8 4 9 Part fuperfluous.
72 11 2 3 true Content add.

78 7 7 0 equal to the Content by the falfe Way.

F. I.
2 2
2 2

4 4
4 4

4 8 4
16 9

75 1 4
3 6 3

Falfe C. 78 7 7

To find how much in Length makes a Foot of any fquared Timber.

Always divide 1728 (the folid Inches in a Foot) by the Area of the Bafe; the Quotient is the Length of a Foot.

This Rule is general for all Timber, which is of equal Thicknefs from End to End, whether it be fquare, triangular, multangular, or round.

Example 1. If a Piece of Timber be 18 Inches fquare; How much in Length will make a Foot folid?

18
18

144
18

324(1728($5\frac{1}{3}$ Inches; which is the Anfwer.
1620

108

By Scale and Compasses.

Extend the Compasses from 1 to 18, that Extent will reach from 18 to 323, the Square or Area of the Base; then extend from 324 to 1728, that Extent will reach down from 1 to 5 Inches, and $\frac{1}{3}$ of an Inch.

Or thus: Extend the Compasses from 18 to 41.569, that Extent, turned twice over from 1, will at last fall upon 5 $\frac{1}{3}$, as before.

Note, That 41.569 is the square Root of 1728.

Example 2. If a Piece of Timber be 22 Inches deep and 15 Inches broad; How much in Length will make a Foot?

```
 22
 15
----
110
 22
----
330)1728(5.23
       ----
       780
       1200
       ----
        210
```

Answer, 5 Inches and .23 Parts.

By Scale and Compasses.

Extend the Compasses from 1 to 15, that Extent will reach from 22 to 330; then extend from 330 to 1728, that Extent will reach from 1 to 5.23 Inches, the Length of a Foot.

There is a Line for this Purpose upon most ordinary Rules, with a little Table at the End of all such Numbers as exceed the Length of the Rule, such as this annexed.

0	0	0	0	9	0	11	3	9	Inches.
144	36	16	9	5	4	2	2	1	Feet.
1	2	3	4	5	6	7	8	9	Side of the Sq

Here you see, if the Side of the Square be 1, the Length must be 144 Feet; if two Inches be the Side of the Square, it must be 36 Feet in Length, to make a solid Foot, *&c.*

If the Side of the Square be not in the little Table, you will find it upon the Line; thus, if the Side of the Square be 16 Inches, you will find it against 6 Inches and 7 Tenths, counted from the other End of the Rule.

Then if you take the Length of a Foot from the Line of Inches with your Compasses, and run the Compasses along the Piece, from End to End, you will find how many Feet are contained in that Piece; or you may cut off any Number of solid Feet that shall be desired; but if the Sides of the Piece be unequal, find a mean proportional Number, as is before taught by dividing the Distance upon the Line of Numbers into two equal Parts: Thus, if the Breadth be 25 Inches and the Depth 9 Inches, divide the Space upon the Line of Numbers into two equal Parts, and you will find the middle Point at 15; so is 15 Inches the geometrical mean Proportional sought; then if you look for 15 upon the Line above-mentioned, you will find 7 Inches and a little above half to be the Length of a Foot.

§ III.

§ III. *Of unequal Squared Timber.*

BY unequal Squared Timber, I mean all ſuch as have unequal Baſes; that is, ſuch as is thicker at one End than at the other; and ſuch are moſt Timber-trees when they are hewn, and brought to their Squares.

The uſual Way to meaſure ſuch Timber, is to take a Square about the Middle of the Piece, which they take to be a mean Square: This Way, when the Piece is pretty near as thick at one End as at the other, is ſomething near the Truth; but when there is a great Diſproportion between the Ends of the Piece, the Error is conſiderable. All ſuch Solids being the Fruſtums of Pyramids, the true Way of meaſuring them muſt be by Sect. VII. Chap. 2. I ſhall give an Example or two, which I will work both by the true and falſe Ways, whereby you will ſee the Difference.

Example 1. If a Piece of Timber be 25 Inches ſquare at the greater End, and 9 Inches ſquare at the leſſer End, and 20 Feet long; How many Feet of Timber are in that Tree?

```
       25
        9
      ---
Sum    34
      ---
Half   17 the Side of the Square in the Middle.
       17
     ----
      119
      17
     ----
     289
      20
    -----
144)5780(40.13
     200
      560
     ----
     128
```

Anſwer, 40.13 Feet, by the falſe Way.

There is a Line for this Purpoſe upon moſt ordinary Rules, with a little Table at the End of all ſuch Numbers as exceed the Length of the Rule, ſuch as this annexed.

0	0	0	0	9	0	11	3	9	Inches.
144	36	16	9	5	4	2	2	1	Feet.
1	2	3	4	5	6	7	8	9	Side of the Sq

Here you ſee, if the Side of the Square be 1, the Length muſt be 144 Feet; if two Inches be the Side of the Square, it muſt be 36 Feet in Length, to make a ſolid Foot, *&c.*

If the Side of the Square be not in the little Table, you will find it upon the Line; thus, if the Side of the Square be 16 Inches, you will find it againſt 6 Inches and 7 Tenths, counted from the other End of the Rule.

Then if you take the Length of a Foot from the Line of Inches with your Compaſſes, and run the Compaſſes along the Piece, from End to End, you will find how many Feet are contained in that Piece; or you may cut off any Number of ſolid Feet that ſhall be deſired; but if the Sides of the Piece be unequal, find a mean proportional Number, as is before taught by dividing the Diſtance upon the Line of Numbers into two equal Parts: Thus, if the Breadth be 25 Inches and the Depth 9 Inches, divide the Space upon the Line of Numbers into two equal Parts, and you will find the middle Point at 15; ſo is 15 Inches the geometrical mean Proportional ſought; then if you look for 15 upon the Line above-mentioned, you will find 7 Inches and a little above half to be the Length of a Foot.

§ III.

§ III. *Of unequal Squared Timber.*

BY unequal Squared Timber, I mean all ſuch as have unequal Baſes; that is, ſuch as is thicker at one End than at the other; and ſuch are moſt Timber-trees when they are hewn, and brought to their Squares.

The uſual Way to meaſure ſuch Timber, is to take a Square about the Middle of the Piece, which they take to be a mean Square: This Way, when the Piece is pretty near as thick at one End as at the other, is ſomething near the Truth; but when there is a great Diſproportion between the Ends of the Piece, the Error is conſiderable. All ſuch Solids being the Fruſtums of Pyramids, the true Way of meaſuring them muſt be by Sect. VII. Chap. 2. I ſhall give an Example or two, which I will work both by the true and falſe Ways, whereby you will ſee the Difference.

Example 1. If a Piece of Timber be 25 Inches ſquare at the greater End, and 9 Inches ſquare at the leſſer End, and 20 Feet long; How many Feet of Timber are in that Tree?

```
       25
        9
       ——
Sum    34
       ——
Half   17 the Side of the Square in the Middle.
       17
      ———
      119
      17
     ————
     289
      20
    —————
144)5780(40.13
     200
      560
     ————
      128
```

Anſwer, 40.13 Feet, by the falſe Way.

By Rule II. Sect. VII. Chap. 2.

25	25
9	9
225	16 Difference of the Sides.
	16
	96
	16
	3)256 the Square.
	85.333
	225
	310.333
	20
	144)6206.660(43.101
	446
	146
	260
	116

Anſwer, 43.101 Feet, by the true Way; ſo that there is near 3 Feet Difference.

By Scale and Compaſſes.

Extend from 1 to 9, that Extent will reach from 25 (the ſame Way) to 225, the Rectangle of the Sides of the two Baſes; then the Difference between the ſaid Sides is 15; extend from 3 to 16, that Extent will reach from 16 to 85.333, a third Part of the Square; which added to 225, the Sum is 310.333, a mean Area: Then extend from 144 to 310.333, that Extent

Extent will reach from 20 (the Length) to 43.1 Feet, the Content the true Way.

Extend the Compasses from 12 to 17 (the Side of the middle Square), that Extent will reach from 20 (the Length, being twice turned over) to 40.1 Feet, the Content by the false Way.

Example 2. If a Piece of Timber be 32 Inches broad and 20 Inches deep, at the greater End, and 10 Inches broad and 6 deep, at the lesser End, and 18 Feet long; How many Feet of Timber are in that Piece?

Rule I. Sect. VII. Chap. 2.

32	6
20	10
640	60

38400 (195.959 mean Proportional.
1   640 the grearer Base.
60 the lesser Base.

29)284
385)2300   895.959 the Sum.
3909)37500 $6\frac{1}{3}$ Height.
39185)231900
391909)3597500   5375.754

144)5375.754(37.33
1055
477
455
23

Add

Add	32 10	20 6	add.
Sum	42	26	Sum.
Half	21	13	half.

```
   21
   13
   --
   63
  21
  ---
  273 Area in the Middle.
   18 Length.
  ----
 2184
 273
 ----
144)4914(34.12
 594
  180
   360
  ----
   72
```

		Feet.
Anfwer.	Content the true Way———	37.33
	Content the falfe Way———	34.12

By Scale and Compaffes.

Extend the Compaffes from 1 to 20, that Extent will reach from 32 to 640, the Area of the greater Bafe.

Then extend from 1 to 10, that Extent will reach from 6 to 60, the Area of the leffer Bafe: Then extend from 1 to 60, that Extent will reach from 640 to 38400, the Product of the two Areas: Find the fquare Root thereof, by dividing the Space between 1 and 38400 into two equal Parts, fo you will find the middle Point at 195.959, the Root fought; which

is a mean Proportional between the greater and leſſer Areas; then add the mean Proportional and two Areas together, and the Sum is 895.959; which multiplied by 6 (a third Part of the Length) by extending from 1 to 6, that Extent will reach from 895.959 to 5375.75. Then extend from 144 to 5375.75, and that Extent will reach from 1 to 37.33 Feet, the true Content.

For the falſe Way, half the Sum of the Breadths is 21, which is the Breadth in the Middle; and half the Sum of the Depths is 13: Extend from 1 to 13, that Extent will reach from 21 to 273, the Area of the middle Baſe: Then extend from 144 to 273, that Extent will reach from 18, the Length, to 34.12, the Content the falſe Way.

§ IV. *Of Round Timber, whoſe Baſes are equal.*

THE uſual Way to meaſure Round Timber-trees, is to girt them about the Middle with a String, and take the fourth Part of that Girth for the Side of a Square, by which they meaſure the Piece of Timber as if it was ſquare.

But that this is an Error, I ſhall make appear as follows. If the Circumference of a Circle be 1, the Area will be .07958; then the fourth Part of 1 is .25, which ſquared makes .0625; this they take for a mean Area, inſtead of .07958: Therefore the true Content always bears ſuch Proportion to the Content found by the aforeſaid cuſtomary falſe Way, as .07958 to .0625; which is nearly as 23 to 18; ſo that in meaſuring by that cuſtomary falſe Way, there is above the one-fifth Part loſt of what the true Content ought to be.

This

This Error, tho' it has been ſo often confuted, yet it is grown ſo cuſtomary in all Places, that there is little Hope of my prevailing with Men that are ſo wedded to it, to embrace the Truth: I ſhall therefore, in the following Examples, ſhew how to work both the true Way, and alſo the falſe or cuſtomary Way.

Example 1. If a Piece of Timber be 96 Inches in Circumference or Girth and 18 Feet long; How many Feet of Timber are contained therein?

A fourth Part of 96 is 24
24
—
96
48
—
576 Area Baſe.
18
—
4608
576
—
144)10368(72
1008
—
288
288
—

Or thus,

F.	I.
2	0
2	0
4	0
18	
72	0

Content the falſe Way, 72 Feet.

Then

Then the true Way.

```
         96
         96
        ----
        576
       864
        ----
       9216
     .07958
      ------
      73728
     46080
    82944
   64512
   --------
   733.40928 the Area by Prob. 5. § IX. Ch. 2.
          18
  -----------
  586727424
  73340928
  -----------
144)13201.36704(91.67 Feet, the true Content.
   ------------
   241
    973
    1096
    ----
      88
```

By Scale and Compasses.

Extend from 12 to 24 (the fourth Part of the Girth) that Extent, turned twice over from 18 Feet (the Length) will at last fall upon 72 Feet, the Content the customary Way.

Extend from 42.54 to 96 (the Girth) that Extent will reach from 18 Feet, turned twice over, to 91.67 Feet, the true Content.

Example

Example 2. If a Piece of Timber be 86 Inches Girth and 20 Feet long; How many Feet are contained therein?

The fourth Part of 86 is 21.5

F.	I.	P.	
1	9	6	
1	9	6	
1	9	6	
1	4	1	6
	0	10	9
3	2	6	3
			20
64	2	5	0

```
      21.5
      21.5
      ----
      1075
      215
     430
     ------
     462.25
         20
     --------
144)9245.00(64.2
     --------
     605
      290
      ---
       20
```

The Content the falſe Way, 64.2 Feet.

By

By the true Way.

```
          86
          86
        ----
         516
        688
        ----
        7396
       .07958
      ------
       59168
      36980
     66564
    51772
    ---------
    588.57368
           20
    ----------
144)11771.47360(81.74
    ---------
    251
    1074
      667
     ----
      91
```

The true Content, 81.74 Feet.

By Scale and Compasses.

Extend from 12 to 21.5, that Extent, turned twice over from 20, will reach at laſt to 64.2 Feet, the Content the falſe Way.

Extend from 42.54 to 86, that Extent, turned twice over from 20, will at laſt fall upon 81.74 Feet, the true Content.

Theſe Cylindrical Proportions may be very eaſily wrought upon the Line of Numbers.

Problem 1. Having the Diameter of a Cylinder in Inches, to find the Length of a Foot.

Suppoſe the Diameter 22.6 Inches.

As 22.6 is to 46.9, ſo is 1 to a fourth Number, and that to the Length of a Foot in Inches 4.3.

Extend the Compaſſes from 22.6 to 46 9, that Extent will reach from 1 to a fourth Number; then turn them over again, and that will reach to 4.3 Inches.

Problem 2. Having the Diameter in Foot-meaſure, to find the Length of a Foot in Foot-meaſure.

Suppoſe the Diameter 1.88 Feet.

Then, as 1 88 is to 1.28, ſo is 1 to a fourth Number: and ſo is that to the Length of a Foot in Foot-meaſure, .358.

Extend the Compaſſes from 1.88 to 1.128, that Extent, turned twice from 1, will reach to .358 Parts of a Foot.

Problem 3. Having the Circumference in Inches, to find the Length of a Foot in Inches.

Suppoſe the Circumference 71 Inches.

Then, as 71 is to 247.36, ſo is 1 to a fourth Number; and ſo is that to the Length of a Foot in Inches, 4.3.

Extend the Compaſſes from 71 to 147.36, that Extent turned twice from 1, will reach to 4.3 Inches, the Length of a Foot.

Problem 4. Having the Circumference in Foot-meaſure, to find the Length of a Foot in Foot-meaſure.

Suppoſe the Circumference 5.92 Feet.

Then, as 5.92 is to 3.345, ſo is 1 to a fourth Number, and ſo is that to the Length of a Foot In Foot-meaſure, .358.

Extend

Extend the Compaſſes from 5.92 to 3.545, that Extent, turned twice over from 1, will fall upon .358 Parts of a Foot.

Problem 5. Having the Diameter in Inches, and the Length in Inches, to find the Content in Inches.

Suppoſe the Diameter 22.6 Inches, and the Length 156 Inches, or 13 Feet.

Then, as 1.128 is to 22.6, ſo is 156 to a fourth Number; and ſo is that to the Content in Inches, 62674.

Extend the Compaſſes from 1.128 to 22.6, that Extent, turned twice from 156, will fall upon 62674 Inches, the Content.

Note, That 1.128 is the Diameter when the Side of the Square is equal to 1.

Problem 6. Having the Diameter in Foot-meaſure, and Length in Feet, to find the Content in Feet.

Suppoſe the Diameter 1.88 Feet, and the Length 13 Feet.

Then, as 1.128 is to 1.88, ſo is 13 to a fourth Number; and ſo is that to the Content in Feet, 36.27.

Extend from 1.128 to 1.88, that Extent, turned twice from 13, will fall upon 36.27.

Problem 7. Having the Diameter in Inches, and Length in Inches, to find the Content in Feet.

Suppoſe the Diameter 22.6 Inches, and the Length 156 Inches.

Then, as 46.9 is to 22.6, ſo is 156 to a fourth Number; and ſo is that to the Content in Feet, 36.27.

Extend from 46.9 to 22.6, that Extent, turned twice from 156, will fall upon 36.27 Feet, the Content.

Note, That 46.9 is the Diameter of a Circle, whoſe Area is 1728.

Problem 8. Having the Diameter in Inches, and Length in Feet, to find the Content in Feet.

Suppose the Diameter 22.6 Inches, and the Length 13 Feet.

Then, as 13.54 is to 22.6, so is 13 to a fourth Number; and so is that to the Content in Feet, 36.27.

Extend from 13.54 to 22.6, that Extent, turned twice from 13, will fall upon 36.27.

Note, That 13.54 is the Diameter of a Circle, when the Area is 144.

Problem 9. Having the Circumference in Inches, and Length in Inches, to find the Content in Inches.

Suppose the Circumference 71, and the Length 156 Inches.

Then, as 3.545 is to 71, so is 156 to a fourth Number; and so is that to 62674, the Content in Inches.

Extend the Compasses from 3.545 to 71, that Extent, turned twice from 156, will fall upon 62674, the Content.

Note, That 3.545 is the Circumference, when the Side of the Square is equal to 1.

Problem 10. Having the Circumference in Feet, and Length in Feet, to find the Content in Feet.

Suppose the Circumference 5.92 Feet, and Length 13 Feet.

Then, as 3.545 is to 5.92, so is 13 to a fourth Number; and so is that to 36.27.

Extend from 3.545 to 5.92, that Extent, turned twice from 13, will fall upon 36.27 Feet, the Content.

Problem

Problem 11. Having the Circumference in Inches, and Length in Inches, to find the Content in Feet.

Suppofe the Circumference 71 Inches, and Length 156 Inches.

Then, as 147.36 is to 71, fo is 156 to a fourth Number; and fo is that to the Content in Feet 36.27.

Extend the Compaffes from 147.36 to 71, that Extent, turned twice from 156, will fall upon 36.27 Feet, the Content.

Note, That 147.36 is the Circumference of a Circle, whofe Area is 1728.

Problem 12. Having the Circumference in Inches, and Length in Feet, to find the Content in Feet.

Suppofe the Circumference 71 Inches, and Length 13 Feet.

Then, as 42.54 is to 71, fo is 13 to a fourth Number; and fo is that to the Content in Feet, 36.27.

Extend the Compaffes from 42.54 to 71, that Extent, turned twice from 13, will reach to 36.27 Feet, the Content.

Note, That 42.54 is the Circumference of a Circle, whofe Area is 144.

§ V. *Of Round Timber, whose Bases are unequal.*

THE usual Way to measure Round Timber (as I said before) is to take a fourth Part of the Girth in the Middle of the Piece, for the Side of a mean Square. But this Way I have proved to be erroneous in Timber that is all the Way of an equal Thickness; and it must be much more so in Timber that is tapering; and the more tapering it is, the greater is the Error: For to the Error in the last Section, there is added the Error in the third Section; therefore, to measure all such Timber according to Art and Truth, such a Piece ought to be considered as a Frustum of a Cone, and should be measured by the Rules given in Section VIII. Chapter 2, by which Rules the following Examples are wrought.

Example 1. If a Piece of Timber be 9 Inches Diameter at the lesser End, and 36 Inches at the other End, and 24 Feet long; How many Feet of Timber are therein?

	36	36	} Subtract.
	9	9	
Rect.	324	27	Difference.
		27	
		189	
		54	
		3)729	the Square.
		243	One-third.
		324	Rectangle add.
		567	

.7854

.7854
.567

54978
47124
39270

A mean Area 445.3218
24

17812872
8906436

144)10687.7232(74.22

607
317
292

4

Answer, 74.22 Feet.

Or thus, by Feet and Inches.

F.	I.			F.	I.		
3	0			2	3		Difference.
0	9			2	3		
2	3	Rect.		4	6		
				0	6	9	
				5	0	9	the Square.
				1	8	3	One-third.
				2	3	0	Rectangle added.
				3	11	3	a mean Square.

Then

Then, As 14 is to 11, ſo is 3 F. 11 I. 3 P. to the Area.

	F.	I.	P.	
	3	11	3	
			11	
7)	43	3	9	
2)	6	2	3	
	3	1	1	6
				6
	18	6	9	0
				4
	74	3	0	0

Here, inſtead of dividing by 14, I divide by 7 and by 2, becauſe twice 7 is 14.

And inſtead of multiplying by 24 Feet, the Length, I multiply by 6 and 4, becauſe 6 times 4 is 24.

By Scale and Compaſſes this is too troubleſome.

Example 2. If a Piece of Timber be 136 Inches Circumference at one End, 32 Inches Circumference at the other End, and 21 Feet long; How many Feet of Timber are contained in that Piece ?

```
 136          136
  32           32
----         ----
 272          104 Difference.
408           104
----         ----
4352          416
             1040
            -----
           3)10816 the Square.
            -----
             3605.333 One-third.
             4352     Rectangle add.
             --------
             7957.333 a mean Cir. squared.
               .07958
             --------
             63658664
            39786665
           71615997
          55701331
          -----------
          633.24456014 the mean Area.
                    21
          ------------
           63324456014
         126648912028
        --------------
        13298.13576294

          144)13298.13(92.34
              ---------
              338
               501
                693
               ----
               117
```

Answer, 92.34 Feet.

By

By Feet and Inches, thus ;

F.	I.			F.	I.			
11	4			8	8	Difference.		
2	8			8	8			
22	8			69	4			
7	6	8		5	9	4		
30	2	8	3)	75	1	4 the Square.		
				25	0	5	4	
				30	2	8	0	
				55	3	1	4	the Sq. of the Circum.

	F.	I.	P.	S.	
88 : 7 : :	55	3	1	4 :	the mean Area.
				7	
11)	386	9	9	4	
8)	35	1	11	9	
	4	4	8	11	the mean Area.
				7	
	30	9	2	5	
				3	
Facit	92	3	7	3	

§ VI.

§ VI. *Of the Five Regular Bodies.*

THESE Bodies may all be measured by the 4th Section of Chap. 2. except it be the Cube, or Hexaedron, which is already measured in Section I. of that Chapter.

1. *Of the* TETRAEDRON.

A Tetraedron is a Solid, contained under four equal and equilateral Triangles.

Let ABCD be a Tetraedron, whose Side is 12 Inches, the perpendicular Height 9.798 Inches.

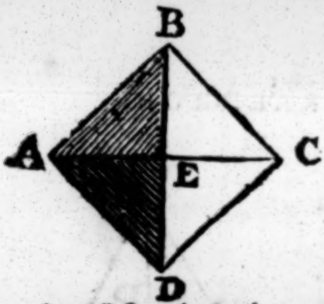
By Sect. V. Chap. 1. the Area of the Triangle will be found 62.352; a third Part of it is 20.784, which multiplied by the perpendicular Height, the Product is 203.641632 solid Inches, the Content.

10.392	the Perpendicular of the Triangle.
6	Half the Side.
62.352	Area of the Triangle.
20.784	one third Part.
9.798	the perpendicular Height.
166272	
187056	
145488	
187056	
203.641632	

The

The ſuperficial Content is four times the Area of the Triangle; *viz.* 249.408 Inches, becauſe there are 4 Triangles.

2. *Of the* OCTAEDRON.



The Octaedron is a Body contained under eight equal and equilateral Triangles.

Let ABCDE be an Octaedron, whoſe Side is 12 Inches; the Content ſolid and ſuperficial is required.

An Octaedron is compoſed of two quadrangular Pyramids joined together by their Baſes; therefore, if the Area of the Baſe be multiplied into a third Part of the Length of both Pyramids, the Product will be the ſolid Content.

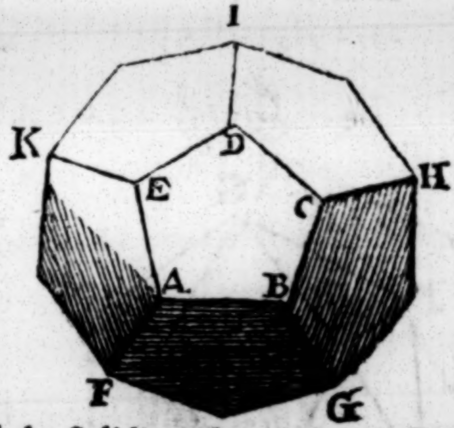
5.6568 a third Part of the Length.
144 Area of the Square Baſe.

```
  226272
 226272
56568
--------
814.5792 the Solidity.
```

The ſuperficial Content will be juſt double to that of the Tetraedron; *viz.* 498.816, becauſe the Side of this is ſuppoſed to be equal to the Side of that, and becauſe the Octaedron is contained under eight Triangles, and the Tetraedron but under four.

3. *Of the* DODECAEDRON.

The Dodecaedron is a ſolid Body, contained under twelve pentangular Planes.



Let ABCDEFG HIK be a Dodecaedron, each Side thereof being 12 Inches; the Content, ſolid and ſuperficial, is required.

The Solidity of the Dodecaedron is compoſed of 12 pentangled Pyramids, whoſe Vertexes all meet in the Center. Therefore, if we find the Solidity of one of thoſe Pyramids, and multiply that by 12, that Product will be the Solidity of the Dodecaedron.

The Altitude of one of the pentangled Pyramids will be found to be 13.36219.

The Perpendicular of the Pentagon will be

8.258292
30 half Sum of the Sides.

247.748760 Area of the Pentagon.
60454.4 a third Part of 13.36219 inverted.

99099504
9909950
1238744
99099
1486

1103.48783 Content of one Pyramid.
12

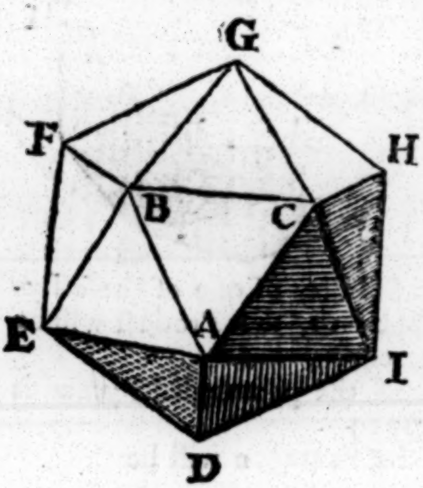
13241.85392 the Solidity of the Dodecaedron.

 If

If the Area of the Pentagon be multiplied by 12, the Product will be the ſuperficial Content.

$$\begin{array}{r} 247.74876 \\ 12 \\ \hline 2972.98512 \end{array}$$ the ſuperficial Content.

4. *Of the* ICOSAEDRON.



The Icoſaedron is a ſolid Body, contained under twenty equal and equilateral Triangles.

Let ABCDEFGHI be an Icoſaedron, each Side thereof being 12 Inches; the Content, ſolid and ſuperficial, is required.

The Icoſaedron is compoſed of twenty triangular Pyramids, with their Vertexes all joining in the Center.

Therefore, if the ſolid Content of one Pyramid be multiplied by 20, the Product is the whole ſolid Content of the Icoſaedron.

10.39224

10.39224 the Perpendicular of the Triangle.
6 half the Side.

62.35344
20

1247.06880

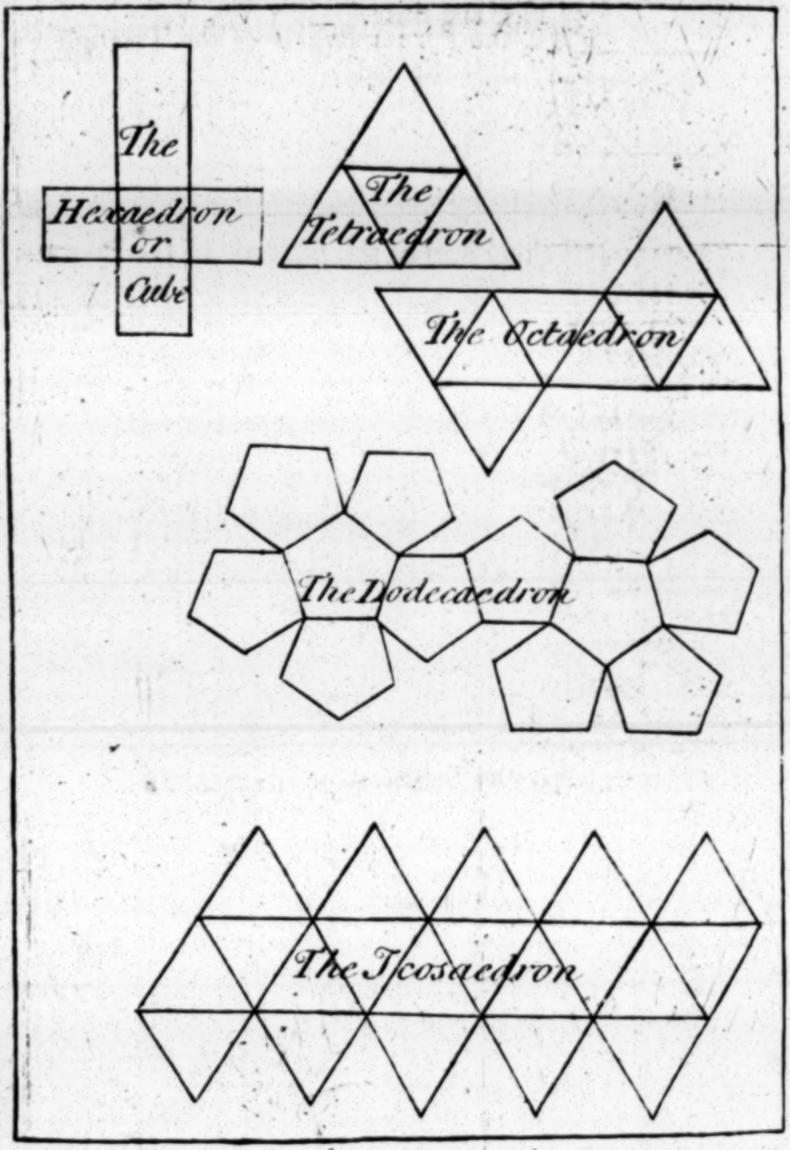
3.0230456 third Part of the Altitude of the Pyram.
44353.26

181382736
6046091
906914
151152
9069
1209
121

188.497292
20

3769.945840 the Solidity.

The superficial Content, 1247.0688



By these Figures you may cut these Bodies in fine Pasteboard, cutting all the Lines half through, and so turn them up and glue them.

A TABLE

A TABLE *ſhewing the Solidity and ſuperficial Content of any of the Regular Bodies, the Side being* 1, *or Unity.*

The Names of the Bodies.	The Solidity.	Superficies.
Tetraedron	0.1178511	1.732051
Octaedron	0.4714045	3.464102
Hexaedron	1.0000000	6.000000
Icoſaedron	2.181695	8.660254
Dodecaedron	7.663119	20.645729

By this Table, the Content, either ſuperficial or ſolid, of any of theſe Bodies, may very readily be found; for all like ſuperficial Figures are in Proportion one to another, as are the Squares of their like Sides: Therefore it will be, As the Square of 1 (which is 1) is to the ſuperficial Content in the Table; ſo is the Square of the Side of the like Body, to the ſuperficial Content of the ſame Body. Therefore, if the Number in the Table be multiplied by the Square of the Side given, the Product is the ſuperficial Content required.

Again; all like Solids are in ſuch Proportion to each other, as are the Cubes of their like Sides. Therefore it will be, As 1 (which is the Cube of 1) is to the ſolid Content in the Table, ſo is the Cube of the Side given, to the ſolid Content required. Therefore, if the Number in the Table be multiplied by the Cube of the given Side, the Product will be the ſolid Content of the ſame Body.

Example 1. If the Side of a Dodecaedron be 12 Inches (as before); What is the Content solid and superficial?

```
7.663119 the tabular Number.
    1728 the Cube of the Side.
--------
 61304952
 15326238
53641833
7663119
------------
13241.869632 the solid Content, nearly the same as (before.

20.645729 the tabular Number.
      144 the Square of the Side.
---------
 82582916
82582916
20645729
-----------
2972.984976 the superficial Content.
```

By Scale and Compasses.

Extend from 1 to 12 (the Side) that Extent being turned three times over from 7.63119, will at last fall upon 13241.86, &c. the solid Content.

And if you apply the same Extent twice from 20.645729, it will at last fall upon 2972.98, &c. the superficial Content.

Example 2. If the Side of an Octaedron be 20 Inches; What is the Content solid and superficial?

.4714045	the tabular Number.
8000	the Cube of the Side.
3771.2360000	the solid Content.
3.464102	the tabular Number.
400	the Square of the Side.
1385.640800	the superficial Content.

By Scale and Compasses.

Extend from 1 to 20, that Extent, turned three times over from .4714045, will at last fall upon 3771.236, the solid Content. The same Extent, turned twice over from 3.464, &c. will at last fall upon 1385.64, the superficial Content.

§ VII. *How to measure any irregular Solid.*

IF you have any Piece of Wood or Stone that is craggy and uneven, and you desire to find the Solidity, put the Solid into any regular Vessel, as a Tub, a Cistern, or the like, and pour in as much Water as will just cover it; then take out the Solid, and measure how much the Fall of the Water is, and so find the Solidity of that Part of the Vessel.

Example

Example. Suppofe a Piece of Wood or Stone to be meafured, and fuppofe a Tub 32 Inches Diameter, into which let the Stone or Wood be put, and covered with Water; then, when the Solid is taken out, fuppofe the Fall of the Water 14 Inches; Square 32, and multiply the Square by .7854, the Product will be 804.2496, the Area of the Bafe; which multiplied by 14, the Depth or Fall of the Water, and the Product is 11259.49, *&c.* which divided by 1728, the Quotient is 6.51 Feet; and fo much is the folid Content required.

CHAP.

CHAP. V.

Practical Questions in MEASURING.

Question 1. IF a Pavement be 47 Feet 9 Inches long and 18 Feet 6 Inches broad, I demand how many Yards are contained therein?

F.	I.		F. I.
47	9		47·75
18	6		18.5
376	0		23875
47			38200
23	10	6	4775
9	0	0	
4	6	0	9)883·375
883	4	6	98.1

Answer, 98 Yards 1 Foot.

Quest. 2. There is a Room, whose Length is 21.5 Feet and the Breadth 17.5 Feet, which is to be paved with Stones, each 18 Inches square; I demand how many such Stones will pave it?

21.5	1.5
17.5	1.5
1075	75
1505	15
215	2.25 Area of one Stone.

2.25)376.25(167

1512
1625

50 Answer, 167 Stones.

Quest. 3. There is a Room 109 Feet 9 Inches about, and 9 Feet 3 Inches high, which is all (except two Windows, each 6 Feet 6 Inches high and 5 Feet 9 Inches broad) to be hung with Tapestry that is Ell broad; I desire to know how many Yards will hang the said Room?

From the Content of the Room, subtract the Content of the Windows, and divide the Remainder by the square Feet in a Yard of Tapestry.

3·75

```
 3.75      109.75 Length.                        5.75
    3       92.5  Breadth.                       6.5
 ----      ------                               -----
11.25      54875                                 2875
          21950                                 3450
         98775                                 ------
         -------                               37.375
         1015.1875  Content of the Room.            2
           74.75    Content of the Wind. ſub.  ------
         ---------                             74.750
  11.25)940.4375(83.59
         --------
          4043
           6687
           10925
         --------
             500
```

Anſwer, 83.59 Yards.

Queſt. 4. If the Axis of a Globe be 27.5 Inches; I demand the Content ſolid and ſuperficial.

```
      3.1416
        27.5
     -------
      157080
     219912
    62832
    --------
    86.39.400 the Circumference.
```

```
         86.394  the Circumference.
           27.5  the Diameter.
        -------
         431970
        604758
       172788
      ---------
    6)2375.8350  the superficial Content.
      ---------
        395.9725 a sixth Part.
            27.5
      ----------
        19798625
       27718075
       7919450
    ------------
     1.889.24375
```

Answer, { 6.3 Feet solid.
16.49 Feet superficial. }

Quest. 5. There is the Frustum of a Globe, the Diameter of whose Base is 24 Inches, and the Altitude thereof is 19 Inches; What is the Content solid and superficial?

Find the Superficies as is directed in Page 188, and 24 the Solidity by the first Theorem in Page 190.

```
  24        .7854       .7854      20
  24          567         400      20
 ---       ------     --------    ---
  96        47124     314.1600    400
 48        54978
 ---      39270
 576     --------
         452.3904 } add.
         314.16   }
         --------
         766.5504  the Curve Superficies.
         452.3904  the Base add.
       ----------
        1218.9408  { the whole superficial Con-
                     tent in Inches.
```

```
12×12=144
        3
      ---
      432
      100 the Square of the Alt. add.
      ---
      532 the Sum.
       10 multiply by the Alt.
      ----
     5320
     5236 multiply.
    -----
    31920
   15960
  10640
 26600
 ---------
 2785.5520 the Solidity in Inches.
```

Quest. 6. If a Tree girt 18 Feet 6 Inches, and be 24 Feet long; How many Tons of Timber are contained in that Tree?

```
    F.  I.
  4)18  6 the Girth.
   -----
     4  7  6 a 4th Part.
     4  7  6
   ---------
    18  6  0
     2  8  4  6
        2  3  9
   ------------
    21  4  8  3   Here I multiply by 6 and by
              6      4, because 6 times 4 is 24.
   ------------
   128  4  1  6
              4
   ------------
4|0)51|3  4  6  0
   ------------
    12 33
```

Anſwer, 12 Tons 33 Feet 4 Inches 6 Parts.

Note, That 40 Feet of Timber is a Ton, and 50 Feet a Load.

Note also, That 4 Feet broad, 4 Feet deep, and 8 Feet long, is a Chord of Fire-wood, that is, 128 cubical Feet.

Quest. 7. There is a Cellar to be dug by the Floor, whose Length is 33 Feet 7 Inches, and the Breadth is 18 Feet 9 Inches, and its Depth is to be 5 Feet 9 Inches; I demand how many Floors of Earth are in that Cellar?

F.	I.		
33	7		the Length.
18	9		the Breadth.
264			
33			
16	9	6	
8	4	9	
9	0	0	
1	6	0	
629	8	3	
5	9	0	the Depth.
3148	5	3	
314	10	1	
157	5	0	
324)3620	8	4(11	
380			
56			

Answer, 11 Floors 56 Feet.

Note, That 18 Feet square and a Foot deep is a Floor of Earth, that is, 324 solid Feet.

Quest.

Quest. 8. There is a Roof covered with Tiles, whose Depth on both Sides (with the usual Allowance at the Eaves) is 35 Feet 6 Inches, and the Length 48 Feet 9 Inches? How many Squares of Tiling are contained therein?

F.	I.	
48	9	
35	6	
240		
144		
24	4	6
17	6	0
8	9	0
17|30	7	6

Answer, 17 Squares 30 Feet.

Quest. 9. There is a Cone, whose Diameter at the Base is 42 Inches, and the perpendicular Height 94 Inches, and it is required to cut off two solid Feet from the Top End thereof; I demand what Length upon the Perpendicular must be cut off?

```
       42        1728          94
       42           2          94
     ----        ----        ----
       84        3456         376
     168                     846
     ----                    ----
     1764                    8836 Square.
     .7854                     94
     ----                    ----
     7056                   35344
    8820                   79524
   14112                   ------
  12348                    830584 the Cube.
  -------
  1385.4456
        94
  ---------
  55417824
 124690104
 ----------
3)130231.8864
 ----------
 43410.6288
```

All the solid Bodies are in triplicate Reason of their homologous Sides, by *Eucl.* 12, 12; 12, 18; and 11, 33: Therefore it will be,

```
Solidity of the Cone.   Cub. Alt.   Solidity of 2 Feet.
43410.6288          :   830584    : :  3456 :
                          3456
                        -------
                        4983504
                       4152920
                      3322336
                     2491752
                    ----------
434106288)2870498304(66124 the Cube of the
                                   Length.
          ----------
          265860576
            5396803
            1055740
             187528                         66124
```

66124(40.43
64

2124000 Resolvend.

12
48

492 Divisor.

120
4800

48120 Divisor.

64
1920
19200

1939264 Subtrahend.

184736000 Resolvend.

1212
489648

4897692 Divisor.

27
10908
1468944

147003507 Subtrahend.

37732493.

Answer, The Length upon the Perpendicular must be 40.43 Inches. If it had been 3 Feet, the Length had been 46.29 Inches.

If two Feet were to be cut off from the Bottom, or greatest End, then from 43410.6288 subtract 3456, and the Remainder is 39954.6288. Then say,

```
43410.6288 : 830584 :: 39954.6288
                           830584
                      -----------
                        1598185152
                       3196370304
                      1997731440
                     11986388640
                    3196370304
                    ----------------
13410.6288)33185675407.2192(764459(91.4
           ------------------ 729
           279823524          ----
            19359751          35459
             1995500         ------
              259075             27
               42022          243
              ------          -----
                2952          2457
                              -----
                                271
                              243
                              ------
                              24571
                              ------
                              10888000
                              --------
                                   273
                                24843
                                ------
                                248703
```

Answer, It must be cut at 91.4 Inches from the Top, or 2.6 Inches from the Bottom.

Quest.

Quest. 10. If a square Piece of Timber be 12 Feet long, and if the Side of the Square of the greater Base be 21 Inches, and the Side of the Square of the lesser Base be 3 Inches; How far must I measure from the greater End, to cut off 3 solid Feet.

First find the Length of the whole Pyramid, thus; the Difference between 21 and 3 is 18: Then,

Diff. Length. great. Length.
As 18 : 12 : : 21 : 14.

So I find the whole Length of the Pyramid to be 14 Feet, or 168 Inches.

The solid Content of the whole Pyramid is 24696 Inches, and the solid Content of 5 Feet is 8640; which subtracted from 24696, there remains 16056 Inches. Then, the Cube of 168 (the Length) is 4741632. Then,

24696 : 4741632 : : 16056 :

3082752, whose Cube Root is 145.54; subtract this Root from 168 (the Length) and there remain 22.46 Inches, which is the Length of 5 solid Feet at the great End.

Quest. 11. Three Men bought a Grinding-stone of 40 Inches diameter, which cost 20 Shillings; of which Sum the first Man paid 9 Shillings, the second 6 Shillings, and the third 5 Shillings; I demand how much of the Stone each Man must grind down, proportionable to the Money he paid?

All Circles are in duplicate Reason of their Diameter, by *Euc.* 12, 2.

Square the Semidiameter, which makes 400. Then

s. Square. s.
20 : 400 : : 9 : 180.

This 180 is the Square of the Semidiameter of the Circle belonging to the first Man.

$$\text{And, } \overset{s.}{20} : 400 :: \overset{s.}{6} : 120$$

This 120 is the Square of the Semidiameter of the Circle belonging to the second.

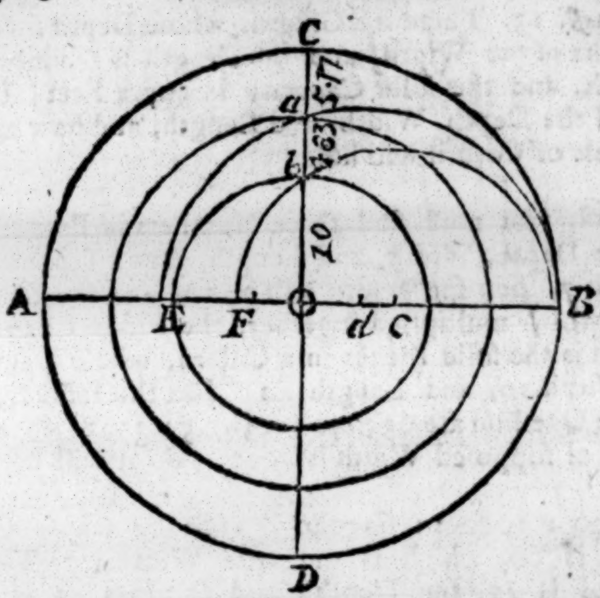
$$\text{And, } \overset{s.}{20} : 400 :: \overset{s.}{5} : 100.$$

This 100 is the Square of the Semidiameter of the Circle belonging to the third.

Then, from 400 (the Square of the Semidiameter of the Stone) subtract 180; and there remains 220, whose square Root is 14.83 Inches; which subtracted from 20 Inches (the Semidiameter), there remain 5.17 Inches, which is the Breadth of the Ring, or Part of the Stone which must be ground down by the first.

Then, from 220 subtract 120, and there remains 100, whose square Root is 10; subtract that from 14 83, and there remain 4.83 Inches, the Breadth of the Ring, or Part to be ground down by the second Man. The third must grind down the Remainder, which is 10 Inches, the square Root of 100.

This Question may very easily and speedily be performed geometrically, as in the annexed Scheme.



First, upon the Centre ⊙ strike the Circle ABCD, and cross it at right Angles with the two Diameters AB and CD: Then divide the Semidiameter A⊙ (which suppose 20) in Proportion to 9s. 6s. and 5s. (the several Sums paid by the three Men) by the Points E and F; so shall AE be 9, EF 6, and F⊙5: Then divide EB into 2 equal Parts in d, and upon d, as a Center, strike the Semicircle EaB; and divide FB into 2 equal Parts in c, and upon c, as a Center, with the Radius cF, strike the Semicircle FbB: So have you the Semidiameter ⊙C divided into three such Parts as the Stone ought to be divided; and Circles, struck thro' those Points, will shew how much each Man must grind for his Share.

Quest. 12. A Gard'ner he had an upright Cone,
Out of which should be cut him a Rolling-stone,
The biggest that e'er it could make:
The Mason he said, That there was a Rule
For such sort of Work, but he had a thick Skull:
Now help him for Pity's Sake.

Answer, It must be cut at one-third Part of the Altitude.

Quest.

Quest. 13. There is a Cistern, whose Depth is seven Tenths of the Width, and the Length is 6 times the Depth, and the solid Capacity is 367.5 Feet; I demand the Depth, Width, and Length, and how many Bushels of Corn it will hold?

First, you must find three Numbers in Proportion to the Depth, Width, and Length, thus: suppose the Depth 7, then the Width will be 10, and the Length 42; which multiplied together, the Product is 2940, which is the solid Inches in a Cistern, whose Depth is 7, Width 10, and Length 42. But the solid Inches in the Question are 635040 ($=367.5 \times 1728$) then the Cube of supposed Width is 1000. So it will be,

2940 : 1000 : : 635040 : 216000, whose Cube Root is 60, which is the true Width; 7 Tenths thereof is 42, the Depth; and 6 times 42 is 252 Inches, the Length; which three Numbers being multiplied together, the Product will be 635040. If these solid Inches be divided by 2150.42, and the Quotient is 295 $\frac{66610}{215042}$ Bushels, or 36 Quarters 7 Bushels 1 Peck 4 Pints. And so much will the Cistern hold.

Quest. 14. Suppose, Sir, a Bushel be exactly round,
Whose Depth being measur'd, 8 Inches is found;
If the Breadth 18 Inches and half you discover,
This Bushel is legal all England over.
But a Workman would make one of another Frame;
Sev'n Inch and a half must be the Depth of the same;
Now, Sir, of what Length must the Diameter be,
That it may with the former in Measure agree?

```
            18.5
            18.5
            ----
             925
           1480
           185
           ------
          342.25  the Square.
           .7854
          ------
          136900
         171125
        273800
       239575
       -----------
       268.803150
                8
       -----------
      2150.425200  the solid Inches in a Bushel.

        7.5)2150.4252(286.72336
                 ...
            ----------
               650
                504
                 542
                  175
                   252
                    270
                    450
                    ---
                    .1.
```

```
                                  . .  . .
.7854)286.72336              (365.0666(19.107
                               1
     51103                   -------
      39793                   29)265
        5236                    261
                             -------
                             381)406
                                 381
                             ---------
                           38207)256600
```

Answer, The Diameter must be 19.107 Inches, if the Depth be 7.5 Inches.

Quest. 15. In the Midst of a Meadow well stored
with Grass,
I took just an Acre to tether my Ass;
How long must the Cord be, that feeding all round,
He mayn't graze less nor more than his Acre of
Ground?

By Problem 10. Section IX. Chap. 1. find the Diameter of a Circle containing an Acre; half that will be the Length of the Cord.

The Work.

```
  660 Feet, the Length of an Acre.
   66 Feet, the Breadth of an Acre.
 ----
 3960
3960
-----
43560 the square Feet in an Acre.
```

```
As 1 : 1.2732 : : 43560
                  43560
                 ------
                 763920
                63660
               38196
              50928
             ----------
              . . .
              55460.5920(235.5 Diameter.
              4           -------
             ---          117.75 half.
          43)154
             129
             ---
         465)2560
             2325
             ----
        4705)23559
             23525
             -----
                34
```

Answer, The Cord must be 117 Feet and 9 Inches.

Quest. 16. A Malster has a Kiln, that is 16 Feet 6 Inches square; but he is minded to pull it down, and build a new one, that may be big enough to dry three times as much at a time as the old one will do; I demand how much square the new one must be?

```
      16 5
      16.5
     -----
       825
      990
     165
    ------
    272.25 the Area of the old one.
         3
    ------
    816.75(28.57
    4
   ----
 48)416
    384
   ----
565)3275
    2825
   -----
5707)45000
     39949
    ------
      5051
```

Answer, The Side of the new one must be 28 Feet and near 7 Inches.

Quest. 16. If a round Cistern be 26.3 Inches diameter, and 52.5 Inches deep; How many Inches diameter must a Cistern be to hold twice the Quantity, the Depth being the same? and how many Ale Gallons will each Cistern hold?

```
   26.3
   26.3
   ----
    789
  1578
  526
  ------
  691.69 Square.
       2
  ------
  1383.38(37.19
  9
  ----
67)483
   469
  ----
741)1438
     741
  ------
7429)69700
     66861
     -----
      2839
```

The Diameter of the greater is 37.19 Inches.

```
           691.69 the Square of the lesser Cis-
            .7854           (tern's Diameter.
           ------
           276676
          345845
         553352
        484183
        ---------
        543.253326 Area of the Base.
              52.5
        ----------
        2716266630
       1086506652
      2716266630
      -------------
      28520.7996150 solid Content in Inches.
      -------------
282)28520.799(101.137 Gallons.
    ---------
     320
      387
      1059
       2139
       ----
        165
```

Note, That 282 solid Inches is an Ale or Beer Gallon, and 231 a Wine Gallon.

And 359.05 is the Square of the Diameter of a Circle that will hold a Gallon of Ale at an Inch deep, and 294.12 for Wine.

You

You may find the Content in Gallons, thus: Divide the Square of the Diameter by 359.05, and multiply the Quotient by the Depth.

```
359.05)1383.38         (3.853
       -------           52.5
        306230        -------
         18990          19265
          1037          7706
                       19265
                      -------
```

The Content of the greater, 202.2825 Gallons.

Quest. 18. If the Diameter of a Cask at the Bung be 32 Inches, and at the Head 25 Inches, and the Length 40 Inches; How many Ale Gallons are contained therein?

```
25           32                        359
25           32                          3
---          --                        ----
125          64                        1077
50           96
---          ----
625          1024 Square of the Bung Diameter.
             1024 the same.
              625 Square of the Head Diameter.
           ------
     1077)2673       (2.48
          ----          40
          5190        -----
           8820       99.20
           ----
            204
```

Answer, 99.2 Gallons.

Otherwise you may find a mean Diameter, and work by Scale and Compasses, thus: Subtract 25 from 32, and there remains 7, which multiplied by .7, the Product is 4.9, which added to 25, the Sum is 29.9. Then extend the Compasses from 18.95 to 29.9, that Extent turned twice from 40 (the Length) will fall upon 99.6 Gallons; something more than before.

Quest. 19. There is a Stone 20 Inches long, 15 Inches broad, and 8 Inches thick, which weighs 217 Pounds; I demand the Length, Breadth, and Thickness of another of the same Kind and Shape, which weighs 1000 Pounds?

The Cube of 20 (the Length) is 8000. Then (by *Euc.* 11. 33)

217 : 8000 : : 1000 : 36870.645, whose Cube Root is 38.28 Inches, the Length of the Stone weighing 2000 Pounds. Then say,

20 : 33.28 : : 15 : 24.96
20 : 33.28 : : 8 : 13.312

Answer,	The Length	33.28	Inches.
	The Breadth	24.96	
	The Thickness	13.312	

Quest. 20. If an Iron Bullet, whose Diameter is 4 Inches, weighs 9 Pounds; What will be the Weight of another Bullet (of the same Metal) whose Diameter is 9 Inches?

The Cube of 4 is 64, and the Cube of 9 is 729. Then (by *Euc.* 12. 18.)

lb. *lb.*
64 : 9 : : 729 : 102.515
lb. *oz.* *dr.*
Answer, It weighs 102 8 4 *ferè.*

Quest.

Quest. 21. There is a square Pyramid of Marble; each Side of its Base is 5 Inches, and the Height thereof 15 Inches, and its Weight is 12 Pounds and a Quarter; I demand the Weight of another like square Pyramid, each Side of whose Base is 30 Inches?

The Cube of 5 is 125, and the Cube of 30 is 27000, Then (by *Euc.* 12. 12.)

lb. *lb.*
125 : 12.25 :: 27000 : 2646

Answer, The Weight is 2646 Pounds.

Quest. 22. There is a Ball or Globe of Marble, whose Diameter is 6 Inches, and its Weight 11 Pounds; What will be the Diameter of another Globe of the same Marble, that weighs 500 Pounds?

The Cube of 6 is 216. Then,
lb. *lb.*
11 : 216 :: 500 : 981.1818.

Whose Cube Root is 21.4 Inches, the Diameter sought.

Quest. 23. There is a Frustum of a Pyramid, whose Bases are regular Octagons; each Side of the greater Base is 21 Inches, and each Side of the lesser Base is 9 Inches, and its Length is 15 Feet; I demand how many solid Feet are contained therein?

4.8284 the tabular Number, Page 88.
237 the Square of a mean Side.

337988	21	12
144852	9	12
96568		
	189	3)144
1144.3308	48	
15		48
	237	
57216540		
11443308		

144)17164.9620(119.2
276
1324
289
1

Answer, 119.2 solid Feet.

Quest. 24. There is a Frustum of a Cone, the Diameter of the greater Base is 36 Inches, and the Diameter of the lesser Base is 20 Inches, and the Length or Height is 215 Inches; I demand the Length and solid Content of the whole Cone, and also the solid Content of the given Frustum?

First, Find the Length of the whole Cone, thus:
From 36
Subtr. 20

16 : 215 : : 36 : 483.75

So the Length of the whole Cone is $483\frac{3}{4}$ Inches.

Then

Then find the Content of the whole Cone.

```
     36                 1017.8784
     36                   52.161
   ------              -----------
    216                10178784
   108                 6107270
   ------               101788
   1296                  20357
   .7854                  5089
   ------              -----------
   5184          1728)164132.88(94.98 Feet.
  6480                 -----------
 10368                  8612
 9072                   17008
 ----------             14568
 1017.8784 Area Base.   ------
                          744
```

Thus I find the Solidity of the whole Cone 94.98 Feet.

Then find the solid Content of the Top-part that is wanting.

.7854

.7854	the Area of Unity.
400	the Square of 20.
3)314.1600	Area of the lesser Base.
104 72	a third Part.
268.75	Altitude of the Top-part.
52360	
73304	
83776	
62832	
20944	

1728)28143.5000(16.28 Feet.
10863
4955
14990
1166

	Feet.
Content of the Whole	94.98
Content of the Top-piece	16.28
Content of the Frustum	78.7

Quest. 25. If the Top-part of a Cone contains 26171 solid Inches, and 200 Inches its Length, and the lower Frustum thereof contains 159610 solid Inches; I demand the Length of the whole Cone, and the Diameter of each Base?

200	159610 } add.
200	26171 }
40000	185781 the Sum.
200	
8000000	

26171

26171 : 8000000 : : 185781 : 56789881, whose Cube Root is 384.3 Inches, the Length of the whole Cone.

Then find the Diameter of the lesser Base, thus:

```
200)26171
   -------
   130.855
         3
   -------
   392.565 Area of the lesser Base.
```

Then, by Prob. 10. Sect. IX. Chap. 1.

```
1 : .12732 : : 392.565
                1.2732
            ----------
                785130
              1177695
             2747955
             785130
            392565
            ----------
            499.8137580(22.35
            4
           ----
          42)99
             84
           ----
         443)1581
             1329
           ------
        4465)25237
             22325
           -------
              2912
```

Again;

	Lesser Len.	Less. Diam.	Gr. Leng.	Gr. Diam.
Again;	200 :	22.35 : :	384.3 :	42.94.

		Inches.
Answer,	The Length of the whole Cone	384.3
	The Diameter of the greater Base	42.94
	The Diameter of the lesser Base	22.35

Quest. 26. There is a Frustum of a Cone, whose solid Content is 20 Feet, and its Length 12 Feet; and the greater Diameter bears such Proportion to the lesser as 5 to 2; I demand the Diameters?

$5 \times 5 = 25$
$2 \times 2 = \ \ 4$
$5 \times 2 = 10$

The Sum 39

3)12

4)20(5 Feet.

These 5 Feet are the Triple of a mean Area.

Then, 1 : 1.27324 : : 5 : 6.3662

So the triple Square of a mean Diameter is 6.3662.

Then, 39 : 6.3662 : : 25 : 4.080897

This 4.080897 is the Square of the greater Diameter, whose Square Root is 2.020123 Feet; which is 24.24147 Inches. Then,

5 : 24.24147 : : 2 : 9.69659

So the greater Diameter is 24.24147, and the lesser Diameter is 9.69659 Inches.

Quest. 27. There is a Room of Wainscot 129 Feet 6 Inches in Circumference, and 16 Feet 9 Inches high (being girt over the Mouldings); there are two Windows, each 7 Feet 3 Inches high, and the Breadth of each, from Cheek to Cheek, 5 Feet 6 Inches; the Breadth of the Shutters of each is 4 Feet 6 Inches; the Cheek-boards and Top and Bottom-boards of each Window,

Window, taken together, is 24 Feet 6 Inches, and their Breadth 1 Foot 9 Inches; the Door-case 7 Feet high, and 3 Feet 6 Inches wide; the Door 3 Feet 3 Inches wide; I demand how many Yards of Wainscot are contained in that Room?

F.	I.	
129	6	
16	9	
782	0	
129		
64	9	
32	4	6
2169	1	6

F.	I.		
7	3		
4	6		
29	0		
3	7	6	
32	7	6	
16	3	9	half.
48	11	3	
		2	
97	10	6	

F.	I.		
3	3		
7	0		
22	9		
11	4	6	half.
34	1	6	

F.	I.	
24	6	
1	9	
4	6	
18	4	6
42	10	6
		2
85	9	0

F.	I.	
7	3	
5	6	
36	3	
3	7	6
39	10	6
		2
79	9	0

F.	I.	
3	6	
7	0	
24	6	} add.
79	9	}
104	3	

The Content of the Room	2169	1	6
The Shutters, at Work and half	97	10	6
The Door, at Work and half	34	1	6
The Cheek-boards, &c.	85	9	0
The Sum	2386	10	6
The Window-lights and Door-case deduct	104	3	0
9)	2282	7	6
	253	5	

Answer, 253 Yards 5 Feet.

Quest. 28. There is a Wall which contains 18225 Cube Feet, and the Height is 5 times the Breadth, and the Length 8 times the Height; What is the Length, Breadth, and Height?

Suppose the Breadth 2, then the Height must be 10, and the Length 80; which three Numbers multiplied together, the Product will be 1600, and the Cube of 2 is 8: then say,

1600 : 8 : : 18225 : 91.125.

Then the Cube Root of 91.125 is 4.5, which is the Breadth; then 5 times 4.5 is 22.5 the Height; and 8 times 22.5 is 180, the Length.

Quest. 29. There is a May-pole, whose Top-end was broken off by a Blast of Wind, and the Top-end, in falling, struck the Ground at 15 Feet Distance from the Foot of the May-pole, the broken Piece was 39 Feet; Now I demand the Length of the May-pole?

By *Eucl.* 1. 47, the Square of the Hypothenuse of a right-angled Triangle is equal to the Sum of the Squares of the Base and Perpendicular.

There-

Therefore, from the Square of 39 subtract the Square of 15, the Square Root of the Remainder is the Piece standing; to which add the Piece broken off, and you have the whole Length.

```
  39      15
  39      15
 ----    ---
 351      75
117      15
 ----    ---
1521     225
 225
 ----
  . .
1296(36
  9
 ----
66)396
   396
  ----
  . . .
```

The Piece standing is	36 Feet.
The Piece broken off is	39 Feet.
The whole Length	75

Question 30.

A May-pole there was, whose Height I would know,
The Sun shining clear, strait to work I did go:
The Length of the Shadow, upon level Ground,
Just sixty-five Feet, when measur'd, I found;
A Staff I had there, just five Feet in Length;
The Length of its Shadow was four Feet One-tenth:
How high was the May-pole, I gladly would know?
And it is the Thing you're desir'd to show.

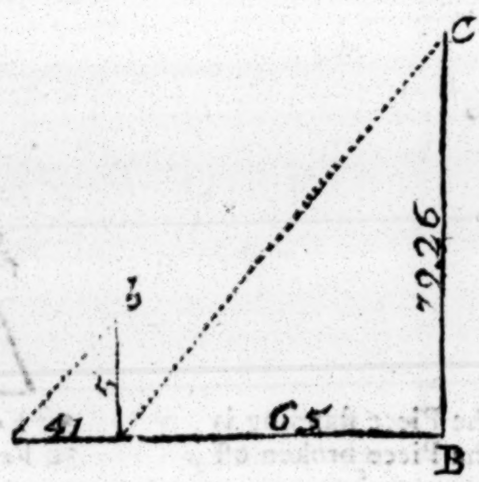
By *Eucl.* 6. 4.

aA : Ab : : AB : BC.

That is,

4.1 : 5 : : 65 : 79.26.

So I find the Height of the May-pole to be 75 Feet and a little above 3 Inches.



Here AB represents the Length of the Shadow of the May-pole, and BC the May-pole; a A the Shadow of the Staff, and A b the Staff.

Quest. 31. What will be the Diameter of a Globe, when the Solidity and superficial Content thereof are equal?

If the Diameter be 1, the Solidity will be .5236, and the Superficies will be 3.1416; that is, as 1 to 6. And to find the superficial Content, we must multiply 3.1416 by the Square of the Axis or Diameter, and the Product is the superficial Content. And for the Solidity, multiply .5236 by the Cube of the Axis, the

the Product is the solid Content; therefore, because .5236 is a sixth Part of 3.1416, we must take 6 for the Diameter sought. For if 3.1416 be multiplied by the Square of 6; *viz.* by 36, the Product will be 113.0976; and if .5236 be multiplied by the Cube of 6; *viz.* by 216, the Product is likewise 113.0976, the Solidity equal to the Superficies.

Therefore, 6 is the true Answer.

Quest. 32. What will the Axis of a Globe be, when the Solidity is in Proportion to the Superficies, as 18 to 8?

Because the Solidity and Superficies is as 1 to 6, when the Axis of the Globe is 1, it will be

$$8 : 18 :: 6 : 13.5.$$

So the Diameter sought is $13\frac{1}{2}$.

If the Proportion of the Solidity to the Superficies had been as 8 to 18, then it will be

$$18 : 8 :: 6 : 2\frac{2}{3}.$$

So then the Diameter will be $2\frac{2}{3}$.

The Reason of these Operations, both in this and the last Question, is from Algebra.

Quest. 33. There are three Grenado-shells, of such Capacity, that the second Shell will just lie in the Concavity of the first, and the third in the Concavity of the second. The Solidity of the Metal of the first Shell is equal to its Concavity, and the Solidity of the Metal to the second, to the Concavity, is as 7 to 5; and the Solidity of the third, or least Shell's Metal, to its Concavity, is as 9 to 4. Now, supposing the Diameter of the first, or greatest Shell, to be 16

 Inches,

Inches, and allowing every ſolid Inch of Iron to weigh 4 Ounces; I demand the Diameter of the two leſſer Shells; and the Thickneſs and Solidity of Metal of every Shell; and alſo the Weight of every Shell?

The Cube of 16 is 4096; then,

1 : .5236 : : 4096 : 2144.6656.

The half thereof is 1072.3328, which is the Solidity of the Metal of the greater Shell, as alſo the Concavity.

.5236 : 1 : : 1072.3328 : 2048.

The Cube Root of 2048 is 12.699, which is the Diameter of the ſecond Shell.

The Sum of 7 and 5 is 12; then,

12 : 5 : : 1072.3328 : 446.805.

This 446.805 is the ſolid Content of the Concavity of the ſecond.

.5236 : 1 : : 446.805 : 853.333.

The Cube Root of 853.333 is 9.485, the Diameter of the leaſt Shell.

The Sum of 9 and 4 is 13; then,

13 : 4 : : 446.805 : 137.47846.

This 137.47846 is the ſolid Content of the Concavity of the third.

.5236 : 1 : : 137.47846 : 262.5639.

The Cube Root of 262.5639 is 6.4034, the Diameter of the leaſt Shell's Concavity.

From	16	the Diameter of the greateſt,
Subtr.	12.699	the Diameter of the ſecond.
Rem.	3.301	

Half is = 1.65 the Thickneſs of Metal of the greateſt.

From

From 12.699 the Diameter of the ſecond,
Subtr. 9.485 the Diameter of the leaſt.

Rem. 3.214

Halfis = 1.607 the Thickneſs of Metal of the ſecond.

From 9.485 the Diameter of the leaſt,
Subtr. 6.403 the Diameter of the Concavity.

Rem. 3.082

Halfis = 1.521 the Thickneſs of Metal of the leaſt.

The Metal of the greateſt is 1072.33 ſolid Inches; which divide by 4 (becauſe every ſolid Inch is a Quarter of a Pound) the Quotient is 268.08 Pounds.

The Metal of the ſecond is 625.52 ſolid Inches; which divided by 4, the Quotient is 156.38 Pounds, the Weight of the ſecond.

The Metal of the leaſt Shell is 309.32 ſolid Inches; which divided by 4, the Quotient is 77.33 Pounds, the Weight of the leaſt.

The Diam. of the	ſecond Shell	12.699	Inches.
	leaſt Shell	9.485	
The Thickneſs of the Metal of the	greateſt	1.65	Inches.
	ſecond	1.607	
	leaſt	1.541	
The Weight	greateſt	268.08	Pounds.
	ſecond	156.38	
	leaſt	77.33	

A SHORT

A SHORT

APPENDIX.

§ I. *Of* GAUGING.

I SHALL not here give the whole Art of Gauging (there being ſeveral Books of that Art already in Print, written by better Hands); but ſhall only lay down ſome ſhort practical Rules, whereby any Artificer, or others, may find the Quantity of Liquor in any Veſſel upon Occaſion.

PROBLEM I.

To find the ſeveral Multipliers, Diviſors, and Gauge-points, belonging to the ſeveral Meaſures now uſed in England.

282)1.0000(.003546 Multiplier for Ale Gallons.
231)1.0000(.004329 Multiplier for Wine Gallons.
268.8)1.000(.0037202 Multiplier for Corn Gallons.
2150.42)1.000(.00046502 Multipl. for Corn Buſhels.

So, if the ſolid Inches in any Veſſel be multiplied by the ſaid Multipliers, the Product will be Gallons in

in the respective Measures; or dividing by the Divisors 282.231, or 268.8, the Quotient will likewise be Gallons.

Note, That 282 solid Inches is a Gallon of Ale or Beer-measure; 231 solid Inches is a Gallon of Wine-measure: 268.8 solid Inches is a Gallon, and 2150.42 solid Inches is a Bushel of Corn-measure

For circular Areas, the following Multipliers and Divisors are to be used.

282).785398(.002785 Multipliers for Ale Gallons.
231).785398(.003399 Multiplier for Wine Gallons.
.785398(282.(359.05 Divisor for Ale Gallons.
.785398(231.(294.12 Divisor for Wine Gallons.
.785398(2150.42(2738 Divisor for Corn Bushels.

The Square Root of the Divisor is the Gauge-point.

The Gauge-point for Squares in	Ale-measure, is	16.79
	Wine-measure, is	15.19
	Malt Bushel, is	46.36
The Gauge-point for circular Figures in	Ale-measure, is	18.95
	Wine-measure, is	17.15
	Malt-bushel, is	52.32

PROBLEM II.

To find the Area in Ale or Wine Gallons, of any rectilineal plain Figure, whether Triangular, Quadrangular, or Multangular.

TO resolve this Problem, you must, by Chap. I. Part II. find the Area in Inches, and then bring it to Gallons, by dividing that Area in Inches by the proper Divisor; *viz.* by 282 for Ale, or by 231 for Wine; or else by Multiplication, by .003.546 for Ale, or by .004329 for Wine; and the Quotient or Product will be the Area.

Example

Example. Suppose a Back or Cooler in the Form of a Parallelogram, or long Square, 250 Inches in Length, and 84.5 Inches in Breadth; What is the Area in Ale or Wine Gallons?

Multiply 250 by 84.5, and the Product is 21125, the Area in Inches, which divide by 282, and the Quotient is 74.9 Gallons of Ale; or multiplied by .003546, the Product is 74.20928 Gallons, nearly the same; and if 21125 be divided by 231, or multiplied by .004329, it will give 91.44 Gallons of Wine.

By Scale and Compasses.

Extend the Compasses from 282 to 250, that Extent will reach from 84.5 to 74.9. And,

Extend from 231 to 250, that Extent will reach from 84.5 to 91.45.

Note, The Areas of all Superficies are always to be understood to be 1 Inch deep; otherwise it could not be said, that the Area of such a Parallelogram Circle, *&c.* is so many Gallons.

Having found the Area of a Back or Cooler, the next thing will be to find out the true Dipping or Gauging-place in that Back, that so the true Quantity of Worts may be computed at any Depth; which may be thus done.

1. When the Bottom of the Back is covered all over (of any Depth) with Worts, or other Liquor, then dip it in eight or ten several Places (more or less, according to the Largeness of the Back), as remote and equally distant from each other as you can well do, noting down the wet Inches and decimal Parts of every Dip.

2. Divide the Sum of all those Dips by the Number of Places you dipped in, and the Quotient will be the mean Wet of all those Dips.

3. Lastly,

3. Lastly, find out such a Place by the Side of the Back (if you can) that just wets the same with that mean Dip, and make a Notch or Mark there for the true and constant Dipping-place of that Back.

Then if any Quantity of Worts (which covers the whole Back) be deeped or gauged at that Place, and the wet Inches so taken be multiplied into the Area of the Back in Gallons, the Product will shew how many Gallons of Worts are in that Back at that time, provided the Sides of the Back do stand at Right-angles with the Bottom.

Problem III.

The Diameter of a Circle being given in Inches, to find the Area thereof in Ale or Wine Gallons.

IF the Square of the Diameter be multiplied by .002785 for Ale, or by .003399 for Wine; or if it be divided by 359.05 for Ale, or by 294.12 for Wine, the Products or Quotients will be the respective Ale or Wine Gallons.

Example. Suppose the Diameter of a Circle be 32.6 Inches; What will be the Area in Ale or Wine Gallons?

The Square of 32.6 is 1062.76.

Then 359.05)1062.76(2.9599 Area in Ale Gallons.
And 294.12)1062.76(3.6133 Area in Wine Gallons.
Or 1062.76×.002785=2.9598 Ale Gallons.
And 1062.76×.0333993=.6133 Wine Gallons.

By

By Scale and Compasses.

Extend the Compasses from 18.95 (the Gauge poin for Ale) to 32.6 (the Diameter) that Extent will reach from 1 to a 4th Number, and from that 4th to 2.9599 Gallons. Or, extend the Compasses from 1, to 32.6, that Extent, turned twice over from .002785, will at last fall upon 2.9599.

For Wine extend from 17.15 (the Gauge-point for Wine) that Extent, turned twice over from 1, will at last fall upon 3.6133 Gallons.

Or thus: Extend from 1 to 32.6, that Extent will reach from .003299, being turned twice over, to 3.6133 Wine Gallons.

PROBLEM IV.

The Transverse (or longest Diameter) and the Conjugate (or shortest Diameter) of an Ellipsis (or Oval) being given, to find its Area in Ale or Wine Gallons.

IF the Rectangle, or Product of the two Diameters, that is, of the Length and Breadth of the Oval, be divided by 359.05, or multiplied by .002785 for Ale, or divided by 294.12, or multiplied by .003399 for Wine, the Quotient or Product will be the Ale or Wine Gallons required.

Example. Suppose the longest Diameter be 81.4 Inches, and the shortest Diameter be 54.6 Inches; What will be the Area of that Oval?

Multiply 81.4 by 54.6, and the Product is 4444.44; then

359.05)4444.44(12.38 Area in Ale Gallons.
294.12)4444.44(15.11 Area in Wine Gallons.
Or 4444.44×.002785=12.38 Ale Gallons.
And 4444.44×.003399=15.11 Wine Gallons.

By Scale and Compasses.

First, find a mean Proportional between 81.4 and 54.6, by dividing the Distance between them into two equal Parts, and the middle Point will be at 66.6, which is the mean Proportional (that is, the Diameter of a Circle equal to the Oval). Then extend the Compasses from 18.95 (the Gauge-point for Ale) to 66.6, that Extent, turned twice over from 1, will at last fall upon 12.38, Ale Gallons: And extended from 17.15 (the Gauge-point for Wine) to 66.6, that Extent, turned twice over from 1, will reach at last to 15.11 Wine Gallons.

PROBLEM V.

To find the Content in Ale or Wine Gallons of any Prism, what Form soever its Base is of.

FIRST, find its solid Content in Inches (by Sect. 1, 2, 3, of Chap. 2. Part II.); then divide that Content in Inches by 282 for Ale, or by 231 for Wine; the respective Quotients will be the Content in Wine or Ale Gallons.

Otherwise, you may find the Content of a Prism by finding the Area of its Base in Gallons (by Problem II. of this Appendix) and multiply that Area by

by the Tun's Height, or Depth within, the Product will be its Content in Gallons.

Example. Suppose a Tun, whose Base is a Parallelogram right-angled, its Length being 49.3 Inches, its Breadth 36.5 Inches, and the Depth of the Tun is 42.6 Inches; the Content in Ale and Wine Gallons is required.

The Length, Breadth, and Depth, being multiplied continually, the Product is 76656.57; which divided by 282, the Quotient is 271.83 Ale Gallons: And divided by 231, the Quotient is 331.84 Wine Gallons: And by dividing by 2150.4 such a Cistern will be found to hold 35.65 Bushels of Corn.

By Scale and Compasses.

Extend the Compasses from 282 to 36.5, the Breadth of the Base, that Extent will reach from 49.3, its Length, to 6.38 Ale Gallons, the Area of the Base; then extend from 1 to 42.6, the Depth, that Extent will reach from 6.38, the Area of the Base, to 271.8 Gallons the Content.

PROBLEM VI.

To find the Content of a Tun, whose Bases are alike and parallel, but unequal, being the Frustum of a Pyramid.

FIND the Area of each Base, and a mean Proportional between them, and multiply the Sum of those three by one third Part of the Depth or Height, and the Product is the Content.

Example. Suppose a Tun, whose Bases are Parallelograms; the Length of the greater is 100 Inches, and its Breadth 70 Inches; the Length of the lesser Base 80, and its Breadth 56; and the Depth of the Tun 42 Inches; the Content in Ale and Wine Gallons is required.

Multiply 100 by 70, the Product is 7000, the Area of the greater Base; and 80 multiplied by 56, the Product is 4480, the Area of the lesser Base; then multiply the two Areas into each other; and the Product is 31360000, whose Square Root is 5600, a geometrical mean Proportional.

The greater Area	7000	add.
The lesser Area	4480	
The mean Proportional	5600	
	17080	
A third of the Depth	14	
	68320	
	17080	

282)239120(847.94 A-g.

231)239120(1035.15 W-g.

PROBLEM

Problem VII.

To find the Content of a Tun, whose Bases are parallel and circular, being the Frustum of a Cone.

YOU may find the Content as in the last Problem, by multiplying the Sum of the Areas of the two Bases, and a mean Proportional, by one third Part of the Depth.

But it will be a shorter Way to find the Area of a mean Circle in Gallons, and multiply that by the Depth, thus: To the Rectangle of the greater and lesser Diameters add one third Part of the Square of the Difference of the Diameters; that Sum is the Square of a mean Diameter, which, divided by 359.05 for Ale, or by 294.12 for Wine, gives the Area of a mean Circle in Ale or Wine Gallons, which, multiplied by the Depth, gives the Content.

Example. Suppose the greater Diameter 80 Inches, and the lesser Diameter 71 Inches, and the Depth 34 Inches, the Content in Ale or Wine Gallons is required.

Multiply 80 by 71, and the Product is 5680; to which add 27 (a third Part of the Square of the Difference of the Diameters) and the Sum is 5707, which is the Square of a mean Diameter; which divide by 359.05, and the Quotient will be 15.895 Gallons the Area; which multiply by 34 (the Depth), and the Product will be 540.43 Gallons, the Content.

By Scale and Compasses.

Add the two Diameters together, and take half the Sum, which is 75.5, which take for a mean Diameter (though it is not exact, yet it will be near enough the Truth, if the Difference between the Diameters be not great); extend the Compasses from 18.95 (the Gauge-point for Ale) to 75.5, the mean Diameter; that Extent will reach from that 34 (the Depth) to a 4th Number, and from that to 540.4 Gallons, the Content.

And if you extend the Compasses from 17.15 (the Gauge-point for Wine) to 75 5, that Extent will reach from 34, twice turned over, to 659.7 Gallons of Wine.

The Method used by the Gaugers for all such Tuns is to take the Diameter in the Middle of every 10 Inches; that is, at five Inches from the Bottom, and at 15, and at 25, *&c.*

Then they find the Area to every one of these Diameters, and enter them in their Books. Then, when they survey, they take the wet Inches and Parts that the Liquor in the Tun is in Depth, and every ten Inches they take the respective Areas, and remove the separating Point one Place toward the Right Hand; and for what odd Inches of the Depth above the even Tens, they multiply the next Area by them, and so add all the several Products together, and the Total will be the Gallons of Liquor in the Tun.

Example. Suppose the Diameter at 5 Inches from the Bottom 64 Inches, and at 15 Inches from the Bottom 67 Inches, and at 25 Inches 70 Inches, and at 35 Inches from the Bottom, the Diameter is 73 Inches. Now the Area answering to 64 Inches is 11.4078 Gallons; and to 67 Inches, is 12.5023 Gallons; and the Area to 70 Inches, is 13.647 Gallons; and to 73, is 14.8418 Gallons: Then, supposing the Depth of the Liquor in the said Tun be found

found to be 3.6 Inches: Now, to cast up this Gauge, first, in the Area answering to 64 Inches, being multiplied by 10, that is by removing the separating Point a Place towards the Right Hand, it will be 114.078 Gallons; and the next will be 125.023; and the next 136.47 Gallons. Now these three will be the Content to 30 Inches deep. Then, to find the Content of the 3.6 Inches, multiply the next Area 14.8418 by 3.6, and the Product is 53.4305: Add all together, and the Sum is the whole Quantity of Liquor in the Tun.

The Content at 10 Inches deep	114.078
The Content at the next 10 Inches	125.023
The Content at the next 10 Inches	136.470
The Content of the 3.6 Inches	53.430
The whole Quantity of Liquor in the Tun	429.001

Problem VIII.

To find the Drip or Fall of a Tun.

SUppose the Tun last mentioned was so placed, that when the Bottom is but just covered on one Side, the Liquor is 4 Inches deep on the Side opposite; How much must be allowed for the Fall of this Tun? That is, How much Liquor is there in the Tun?

The Diameter in the Middle of 4 Inches from the Bottom, is 61.6 Inches; and the Area answering thereunto is 10.568; which multiplied by 2 (that is, half 4), the Product is 21.136 Gallons; and so much Liquor will just cover the Bottom.

But,

But, suppose it was set so much on one Side, as to be 30 Inches deep on one Side, when the Liquor on the opposite Side just cuts between the Bottoms and Staves; How much Liquor will there be in the Tun?

Square the Bottom Diameter, and multiply that Square by the Top Diameter, and divide the last Product by the Sum of the Diameters, and to the Quotient add the Square of the Bottom Diameter, and divide the Sum by 1077.15 for Ale, or by 882.36 for Wine; multiply the Quotient by the Depth, the Product is the Content.

The Bottom Diameter of the fore-mentioned Tun is 61 Inches; and the Diameter, at 30 Inches from the Bottom, is 71.5 Inches; the Square of 61 is 3721; which multiplied by 71.5, the Product is 266051.5; this divided by 132.5 (the Sum of the Diameters) the Quotient is 2007.936: to which add 3721 (the Square of 61), and the Sum will be 5728.936; this divided by 1077.15, the Quotient is 5.3186; which multiplied by 30, the Depth, the Product is 159.558, the Gallons of Liquor in the Tun.

When the Frustum of a Cone or Pyramid is cut, by a diagonal Plane, through the Extremities of the Diameters, as the Liquor in the Tun represents, such Solid is called a Hoof. Vide *Ward's Young Mathematician's Guide*, page 414.)

If it be the Hoof of a square Frustum, instead of dividing by 1077.15, divide by 846 for Ale, or by 693; for Wine. All the rest of the Work is the same.

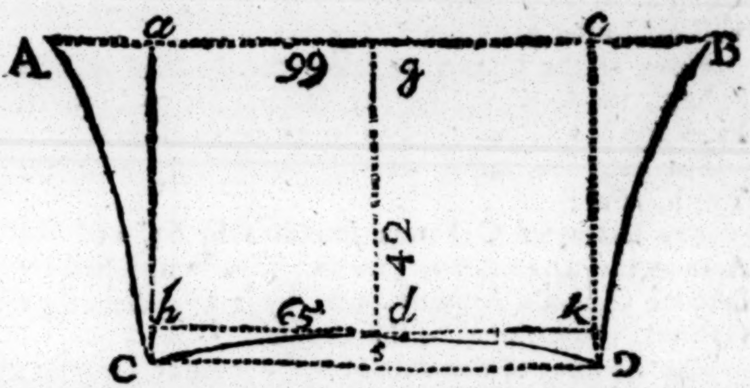
PROBLEM

PROBLEM IX.

To gauge a Copper.

LET ABCD be a ſmall Copper to be gauged.

Take a ſmall Cord of Packthread, make one End faſt at A, and extend the other to the oppoſite Side of the Copper at B, where make it faſt, or cauſe ſome Perſon to hold it very ſtrait; then ſet one End of the Inſtrument in the Bottom of the Copper at C, and move it to and fro, till you find the neareſt Diſtance to the Thread (as at *a*): This Diſtance, *a*C, is the Depth of the Copper, which ſuppoſe to be 47 Inches.



In like manner, ſet the End of the Rule upon the Top of the Crown at *d*, and take the neareſt Diſtance to the Thread, as *dg*, which ſuppoſe 42 Inches: this ſubtracted from *a*C, 47, the Remainder 5 is the Altitude of the Crown.

To find CD, the Diameter of the Bottom of the Crown.

Meaſure AB, the Diameter of the Top, which admit to be 99 Inches; then hold a Thread ſo as a Plummet at the End thereof may hang juſt over C, by

by which means you will find the Diſtance A *a*. Do the like on the other Side; ſo will you find alſo the Diſtance, B; which ſuppoſe 17.5 Inches each; add theſe two together, and ſubtract their Sum (*viz.* 35.) from 99, and the Remainder is 64 Inches, the Diameter at the Bottom of the Crown. The Diameter which touches the Top of the Crown, may be found by the Sliding-rule to be 65 Inches.

Now to find the Content of the Copper from the Crown upwards, that is, the Part AB*kb*, the Depth *gd* being 42 Inches, you may take the Diameter in the Middle of every 6 Inches of the Depth, which ſuppoſe to be as in the ſecond Column of the following Table, the Numbers in the third Column are the reſpective Areas in Ale Gallons, found by Problem III. the fourth Column ſhews the Content of every 6 Inches; all which being added together, the Sum will be the Content of that Part, AB*kb*; that is, ſo much as it will hold after the Crown is covered.

Now, if the Crown be taken for the Fruſtum of a Sphere, the Content (by the latter Part of Sect. II. Page 190) will be found to be 28.75 Gallons.

But may be more readily found, very near the Truth, thus;

The Diameter CD was found to be 64, and the Area to this Diameter is 11.408; this, multiplied by half the Crown's Altitude, *viz.* by 2.5, gives 28.52 Gallons, the Content of the Crown.

The Content of the Part *bk*DC is 57.935 Gallons; from which ſubtract the Content of the Crown, 28.52, and the Remainder is 29.415 Gallons, and ſo much Liquor will juſt cover the Crown.

Part

Parts of the Depth.	Diameter.	Areas.	Content of every 6 Inches.
6	95.3	25.2945	151.767
6	90.1	22.6095	135.657
6	85.0	20.1223	120.734
6	80	17.8246	106 947
6	75.2	15.7499	94.499
6	70 5	13.8426	83.056
6	66	12.1319	72.791
The Sum — —			765.451
To just cover the Crown —			29 415
The whole Content — —			794.866

By Scale and Compasses.

You may find the Areas answering to every one of the Diameters, thus:

Extend the Compasses from the Gauge-point to the Diameter; that Extent, being turned twice over from 1, will at last fall upon the Area of that Circle: Or, being turned twice over from 6, will give the Content of that 6 Inches of the Depth.

Example. Extend the Compasses from 18.95 (the Gauge point) to 95.3; that Extent, turned twice over from 6, will at last fall upon 151.76 Gallons, the Content of the first 6 Inches. And so of the rest.

PROBLEM

Problem X.

To compute the Content of any close Cask.

IN order to perform this difficult Part of Gauging, the three following Dimensions of the Cask must be truly taken;

Viz. { The Bung-diameter, The Head-diameter, The Length of the Cask, } within the Cask.

In taking these Dimensions, it must be carefully observed,

1. That the Bung-hole be in the Middle of the Cask; also, that the Bung-staff, and the Staff opposite to the Bung-hole, are both regular and even within.

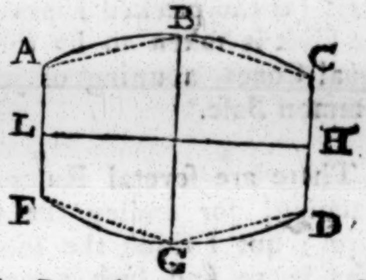
2. That the Heads of the Cask are equal, and truly circular; if so, the Distance between the Inside of the Chine to the Outside of its opposite Staff will be the Head-diameter within the Cask, very near.

3. With a sliding Pair of Calipers (made for that Use) take the shortest Distance, or Length, between the Outsides of the two Heads; from that Length subtract $1\frac{1}{5}$ Inch (more or less according to the Largeness of the Cask) for the Thickness of the Head: The Remainder will be the Length of the Cask within. But if the Cask be empty, you may take the Length, by putting a strait Rod in at the Tap-hole, and allow for the Thickness of the Head.

Now by these Dimensions, one would think the Content of the Cask was perfectly limited; but it will be easy to perceive, by the following Figure, that

the Diameters and Length of one Cask may be equal to those of another, and yet one of those Casks may contain several Gallons more than the other.

As for Instance, the Figure ABCDF is supposed to represent a Cask: Then it is plain, that if the outward curve Lines, A B C, and FGD, are the Bounds or Staves of the Cask, it must needs hold more than if the



inner pricked Lines were the Bounds, or Staves; and yet the Bung-diameter BG, and Head-diameters CD and AF, and the Length LH, are the same in both those Casks.

Whence it appears, that no one general Rule can be given, whereby the Content of all Sorts of Casks can be gauged: And therefore Gaugers do usually suppose every Cask to be in some of these Forms:

1. The middle Frustum of a Spheroid.
2. The middle Frustum of a parabolic Spindle.
3. The lower Frustums of two equal parabolic Conoids.
4. The lower Frustums of two equal Cones.

1. If the Staves of a Cask be very much curved (as the outward Lines of the last Figure), then the Cask is supposed to be the middle Frustum of a Spheroid.

2. If the Staves (between the Bung and the Head) be something less curved, then the Cask is taken to be the middle Frustum of a parabolic Spindle.

3. If the Staves (between the Bung and Head) be very little curved, then the Cask is taken to be the lower Frustums of two equal parabolic Conoids,

abutting or joining together upon one common Base.

4. If the Staves between the Bung and Head be ſtrait (as the pricked Lines in the laſt Figure), then the Caſk is taken to be the lower Fruſtums of two equal Cones, abutting or joining together upon one common Baſe.

There are ſeveral Rules laid down in Books of Gauging, for finding the Content of each ſeveral Form; but I think the ſhorteſt and moſt practical Way is, to find ſuch a mean Diameter, which will reduce the propoſed Caſk to a Cylinder: Thus,

Multiply the Difference of the Bung and Head Diameters by .7 for the Spheroid; by .65 for the ſecond Form, by .6 for the third Form, and by .55 for the fourth Form; and add the Product to the Head-diameter, and the Sum is the mean Diameter.

Example. Suppoſe the Bung-diameter be 32 Inches, the Head diameter 24 Inches, and the Length 40 Inches; the Content in each Variety is required.

The Difference between the Bung and Head-diameter is 8; which multiplied by .7, the Product is 5.6; which added to the Head-diameter, the Sum is 29.6, the mean Diameter: The Area anſwering thereunto will be found (by Prob. III.) to be 2.44 Ale Gallons; which multiplied by the Length, the Product is 97.4 Gallons; and ſo much is the Content, if it be the firſt Form.

Again; if the Difference of the Diameters 8 be multiplied by .65, the Product will be 5.2; which added to the Head-diameter, the Sum is 29.2, for the mean Diameter; and the Area anſwering thereunto is 2.3746 Gallons; which multiplied by 40 (the Length) the Product is 94.98 Gallons, the Content, if it be of the ſecond Form.

Again:

Again; if the Difference 8 be multiplied by 6, the Product is 4.8; which added to the Head-diameter, the Sum is 28.8, the mean Diameter: the Area thereunto is 2.31 Gallons; which, multiplied by 40, gives the Content 92.4 Gallons, for the third Form.

Again; the Difference 8, multiplied by .55, the Product is 4.4; which added to the Head-diameter, makes the mean Diameter 28.4; the Area thereof is 2.2463; which multiplied by 40, the Product is 89.85 Gallons, for the fourth Form.

By Scale and Compasses.

Extend the Compasses from the Gauge-point 18.95 to the first mean Diameter 29.6; that Extent will reach from the Length 40 to a fourth Number, and then to the Content, 97.4 Gallons.

Again; extend from 18.95 to 29.2 (the second mean Diameter) that Extent, turned twice over from 40, will at last fall upon 94.98 Gallons.

Again; extend from 18.95 to 28.8 (the third mean Diameter) that Extent, turned twice over from 40, will at last fall upon 92.4 Gallons.

Again; extend from 18.95 to 28.4 (the fourth mean Diameter) that Extent, turned twice over from 40, will at last fall upon 89.85 Gallons.

Altho' I have all along made use of the Line of Numbers upon the common Two-foot or Eighteen-inch Rules, for the Reason mentioned in the *Preface*; yet the Rules may easily be applied to the Sliding-Rule, thus: To find the Area of a Circle in Gallons, set the Gauge-point upon D (that is, a single Line of Numbers) to 1 upon C (that is, a double Line);

 then

then againſt any Diameter upon D, is the Area upon C, thus;

To find the Content of the Caſk, laſt-mentioned, the firſt Form.

Set the Gauge-point 18.95 upon D, to the Length 40 upon C; then (againſt the mean Diameter) 29.6 upon D, is 97.4 Gallons, the Content upon C.

And againſt 29.2 (the next mean Diameter) on D, is 94.98 Gallons on C.

And againſt 28.8 (the next mean Diameter) on D, is 92.4 Gallons on C.

And againſt 28.4 (the laſt mean Diameter) on D, is 89.85 Gallons on C.

All done without removing the Slider.

A TABLE *of the Segment of a Circle, whose Area is Unity.*

V.S.	Segm	V.S.	Segm	V.S.	Segm.	V.S.	Segm.
1	.0017	99	.9983	26	.2066	74	.7924
2	.0048	98	.9952	27	.2178	73	.7832
3	.0087	97	.9913	28	.2292	72	.7708
4	.0134	96	.9866	29	.2407	71	.7593
5	.0187	95	.9813	30	.2523	70	.7477
6	.0245	94	.9755	31	.2640	69	.7360
7	.0308	93	.9692	32	.2759	68	7241
8	.0375	92	.9625	33	.2878	67	.7122
9	.0446	91	9554	34	2998	66	.7002
10	.0520	90	.9480	35	3119	65	6881
11	.0598	89	.9402	36	.3241	64	.6750
12	.0680	88	.9320	37	.3364	63	6636
13	.0764	87	9236	38	.3487	62	.6513
14	.0851	86	9149	39	.3611	61	.6389
15	-0941	85	.9059	40	.3735	60	6265
16	.1033	84	.8967	41	.3860	59	.6140
17	.1127	83	.8873	42	.3986	58	.6014
18	.1224	82	8776	43	.4112	57	5888
19	.1323	81	.8677	44	.4238	56	.5762
20	.1424	80	.8576	45	.4364	55	.5636
21	.1526	79	.8474	46	4491	54	.5509
22	.1631	78	.8369	47	.4618	53	.5382
23	.1737	77	.8263	48	.4745	52	5250
24	.1845	76	.8155	49	.4873	51	.5127
25	.1955	75	.8045	50	.5000	50	.5000

The Use of the Table of Segments

Is to find the Ullage, or Quantity of Liquor remaining in a Cask, whose Axis is parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Cask.

The RULE *is;*

To the wet or dry Inches of the Bung-diameter, add a competent Number of Cyphers; then divide it by the whole Diameter, the Quotient found in the Table under the Title V.S. gives a Segment; which multiplied by the whole Content of the Cask, the Product shews the Quantity of Liquor in the Cask, if the Dividend was the wet Inches, or the Ullage, if it was the dry.

Let there be a Cask in Form of a Cylinder, whose Bung-diameter is 29 Inches, the dry Part 13, and the wet 16, and the Content 80 Gallons; How many Gallons are wanting to fill the Cask?

Divide the dry Inches 13, by 29 the Bung-diameter, and the Quotient is .448; find the two first Figures .44 under V.S. and the Segment against it is .4238; to which add a proportional Part for the 8, and the whole Segment will be .4333; which multiplied by the Content of the Cask, the Product will be 34.664 Gallons; and so much the Cask wants of being full.

Note, If the Cask be in the Form of a Cylinder, or near that Figure, the Table will give the Ullage exact enough; but if it be a spheroidal Cask, then use the following Method.

1. By

1. By the Bung and Head-diameter, find such a mean Diameter as, you judge, will reduce the proposed Cask to a Cylinder, and then find its Content.

2. From the Bung-diameter subtract the mean Diameter, and take half the Difference.

3. From the wet Inches subtract the said Half-difference; reserve this Difference, then use the Proportion:

As the mean Diameter is to 100
(the Diameter of the tabular Circle),
So is the reserv'd Difference,
to a versed Sine in the Table.

Then, if the tabular Segment be multiplied into the Content (as before) the Product will be the Quantity of Liquor in the Cask.

Example. Let the Cask be the same as in Page 325, of the first Form, where the Bung-diameter is 32 Inches, and the mean Diameter 29.6, and the Content 97.4 Gallons; and suppose the wet Inches 19, to find the Quantity of Liquor in the Cask.

From	32	From	19	
Subtr.	29.6	Subtr.	1.2	
Rem.	1.4	Rem.	17.8	reserved.
Half	1.2			

29.6 : 100 : : 17.8 : .60, the V. S.

The Segment to 60 is .6265, which multiplied by 97.4, the Content, the Product is 61 Gallons, the Quantity of Liquor in the Cask.

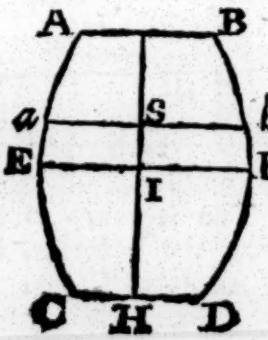
If the dry Inches had been given, by the same Method, you might have found the Ullage, or what the Cask wanted of being full.

To find what Quantity of Liquor is in a Cask, when its Axis is perpendicular to the Horizon; *viz.* when it stands upright upon one of its Heads.

To do this, you must know how to calculate the Area of any Circle, between the Bung and Head, whose Distance from the Bung, or Middle of the Cask, is given; which may be done by this Proportion.

As the Square of half the Length of the Cask is to the Difference between the Bung and Head-areas; so is the Square of any Circle's Distance from the Bung, to the Difference between the Bung-area and the Area of that Circle; *viz.* the Area of the Liquor's Surface.

Then, from the Bung-area, subtract one third Part of the aforesaid Difference; *viz.* between the Bung-area and the Area of the Liquor's Surface: Multiply the Remainder by the Liquor's Distance from the Bung, and the Product will shew what Quantity of Liquor is either above or under half the Content of the Cask.



Example. Let us again suppose the Cask, in Page 325, whose Length is 40 Inches, Bung-diameter 32, and Head-diameter 24, and suppose the wet Inches, SH, 26 Inches.

The Sq. of half the Length is 400, the Distance of the Liquor's Surface from the Bung SI is 6, whose Square is 36; the Area of the Bung D 2.8519 Ale Gallons, and the

the Area of the Head D 1.6042; the Difference 1.2477. Then,

400 : 1.2477 : : 36 : .0751

One third is = .0250

From	2.8519	Bung-area,
Subtr.	.0250	a Third of the Difference.
Rem.	2.8269	
	6	mult. Dist. from the Bung.
	16.9614	Content above the Bung.
Add	48.7	half the Content of the Cask.
	65.66	the Quantity of Liquor in the C.

Problem IX.

Gauging of Malt.

To find the Quantity of Malt in a Cistern, or upon a Floor.

First, Find the Area of the Base in Bushels, by multiplying the Length by the Breadth, and dividing the Product by 2150.42, or only by 2150; and multiply that Area by the mean Depth (How to take the mean Depth, see Problem II.). If the Base be circular or oval, divide by 2731 (see Problem I.)

Example.

Example. There is a Ciſtern, whoſe Length is 84 Inches, and Breadth 54 Inches, and the mean Depth is 43.6 Inches; What is the Content?

Multiply 84 by 54, and the Product is 4536; which divide by 2150, and the Quotient is 2.1097 Buſhels, the Area of the Bottom at 1 Inch deep; which multiplied by the Depth 43.6, the Product is 91.98 Buſhels, the Content.

Example. Suppoſe a Quantity of Malt upon a Floor, whoſe Length is 245 Inches, and the Breadth 184 Inches, and the mean Depth 5.6 Inches; How many Buſhels are there?

Multiply 245 by 114, and the Product is 45080; which divided by 2150, the Quotient is 20.967, the Area of the Baſe; which multiplied by the mean Depth, the Product 117.4 Buſhels, the Content.

By the Sliding-Rule.

There is an inverted Line of Numbers upon ſome Sliding-Rules, marked with the Letter M, which was contrived purpoſely for Gauging of Malt; and there is a double Line of Numbers upon the Rule, and upon the Slider two double Lines of Numbers; all of theſe are of equal Radius, and all work together at once: Thus ſet the Length and Breadth againſt one another upon the inverted Line, and that which ſlides by it; then, on the other Edge of the Rule againſt the Depth, you will find the Content in Buſhels. Thus, in the firſt Example, ſet 54 upon the Slider againſt 84 upon the inverted Line; and then, againſt 43.6 upon the other Part of the Rule, is 91.98 upon the Slider.

Again; in the ſecond Example, ſet 184 upon the Slider to 245 upon the inverted Line; and againſt 5.6 upon the other Part of the Rule, is 117.4 upon the Slider.

§ II.

§ II. *Of* LAND-MEASURING.

I SHALL not here give the whole Art of *Surveying*, but ſuch practical Rules only as may be uſeful to the Country Graſiers and Farmers, whereby they may find the true Content of any Piece of Land, and that by the Chain only (and for want of that, with a Pole or Stick of half a Rod in Length).

PROBLEM I.

To find the Content of a Piece of Land in the Form of a right-angled Parallelogram, or long Square, or what is ſomething near that Form.

TO know whether any Angle in the Field be a Right-angle, or not, you may take a Piece of Board about 4 or 5 Inches broad and an Inch thick, either round or ſquare; and, with a Saw, cut two Kerfs, croſſing each other at Right-angles; and bore a Hole in the Middle of the Back-ſide, to put it upon the End of a Stick. This will repreſent the Inſtrument called a Croſs.

Suppose you would observe the Angle A, to know whether it be a Right-angle (or near thereunto); prick up your Stick, with the Cross upon it, a little Distance from the Fence, as at *a*; and having set up two Marks, as at *b* and *c*, of equal Distance from the Fence, turn one of the Slits directly towards *b*; and then, if the other be directly pointing to *c*, it is a Right-angle.

To measure such a Piece of Ground as this Figure above: If you measure round, and add the opposite Sides together, and take half the Sum (if they be not equal); or else measure down about the Middle of the Length, and Middle of the Breadth; thus, the Side AB being measured, it will be 5.60 (that is, 5 Chains and 60 Links); and the opposite Side CD is 5 Chains 82 Links; the half Sum thereof is 5.71: And the Side BD is 10.38; and the Side AC 10.22; and the half Sum thereof is 10.30 (it will be the same thing, if you measure about the Middle of the Length and Middle of the Breadth); then multiply this mean Length and mean Breadth together; *viz.* 10.30, by 5.71, and the Product is 58.8130; which divide by 10, (because 10 Square Chains is an Acre) by removing the separating Point one Place towards the

 Left-Hand,

Left Hand, and it will be 5.88130; that is, 5 Acres and .88130 Parts; which multiply by 4, and prick off 15 Places, and it will be 3.52520; which 3 towards the Left Hand are 3 Rods; then multiply the decimal Parts by 40, and prick off 5 Places, and it will be 21.00800; which 21 towards the Left Hand are 21 Perches.

	A.	R.	P.
So the whole Content is ——— ———	5	3	21

See the Work.

```
  5.71
 10 30
 -----
 17130
 571
 -----
5.88130
      4
 -----
3 52520
     40
 -----
21.00800
 -----
```

A.	R.	P.
5	3	21

Note. The Chain here made use of, is 4 Poles, or Rods, in Length; the whole Chain being 100 Links.

But, because every Man, that may have Occasion to measure a Piece of Land, can't procure a Chain, I will therefore shew how you may measure a Piece of Land only with a Stick of half a Rod in Length; that is, 8 Feet and 3 Inches; which Stick divide into five equal Parts, so will the whole Rod be divided into ten Parts, and will thereby be adapted to Decimal Arithmetick.

But, becauſe each of thoſe Parts of the Stick are ſomething large (each Part being 19 Inches and 8 Tenths) it will be neceſſary to take your Dimenſions to half of one of thoſe Parts; and then, for that half Part, ſet 5 in the Place of Seconds, thus, ſuppoſe 3 Parts and a half, ſet it down thus, .35.

PROBLEM II.

LET us ſuppoſe a Field in the Form of a long Square, whoſe Length is 45 Rods 5 Parts and a half, and the Breadth 31 Rods 4 Parts and a half; What is the Content?

Multiply the Length and Breadth together, and divide the Product by 160 (becauſe 160 Square Rods are an Acre) and the Quotient is Acres.

$$\begin{array}{r} 45.55 \\ 31.45 \\ \hline 22775 \\ 18220 \\ 4555 \\ 13665 \\ \hline 1432.5475 \end{array}$$

160

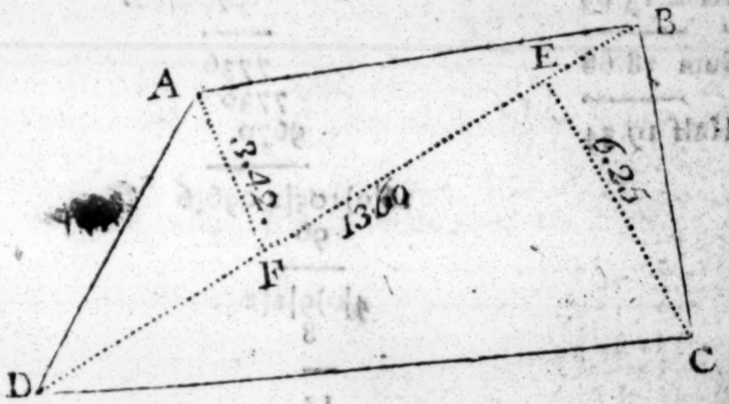
16|0)143|2(8   A. R. P.
128   *Facit* 8 3 32

4|0) 15|2(3
12

32

PROBLEM III.

SUppose a Piece of Ground in the Form of a Trapezium; the Diagonal B D 13 Chains 60 Links, the Perpendicular C E 6 Chains 25 Links, and the Perpendicular A F 3 Chains 42 Links; What is the Content?



Multiply the Diagonal by half the Sum of the Perpendiculars. See Sect. VI. of Chap. I. Part II.

CE=6.25
AF=3.42

Sum 9 67

Half 4.83

13 60=BD
4.83

4080
10880
5440

6.56880
4

2 227520
40

11.00800

A. R. P.
Facit 6 2 11

By Rods, thus;

CE=25 Rods.
AF=13.68

Sum 38.68

Half 19.34

19.34
544=BD

7736
7736
9670

16|0)105|2.096(6
96

4|0)9|2(2
8

12

A. R. P.
Facit 6 2 12

To take the Dimensions of the Field.

Begin at the Angle B, and measure in a direct Line towards D; but when you come at E, set up your Cross, and direct one of the Slits to D, and then look through the other Slit, and if it exactly hits the Angle C, then are you just in the Place where the Perpendicular will fall; but if it does not exactly hit the Point, move backwards and forwards till it does so; then measure the Perpendicular, and set down the Chains and Links, or the Rods and Parts; then continue your Measure towards D; but when you come to F, set up your Cross, and try (as is above directed), whether you be in the Place where the Perpendicular will fall. Then measure the Perpendicular AF, and set down the Chain and Links, or Rods and Parts; then continue your Measure to D, and set down the Measure of the whole Diagonal. This Way of Measuring is very exact and true; but the common Way used by the Graziers and Farmers, is to measure round the Field, and to take half the Sum of the opposite Sides for a mean Side; but the last-mentioned Piece of Ground, being measured so, will come to

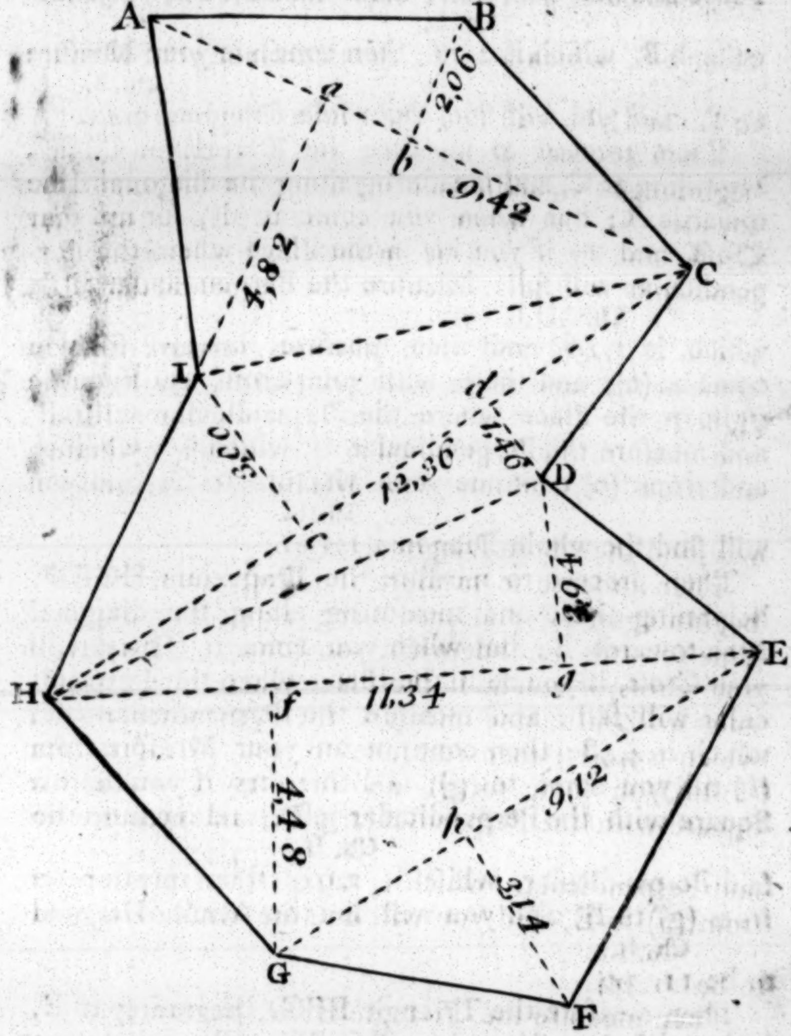
A. R. P. R P.
7 0 22, which is 2 10 more than the Truth.

Problem IV.

How to meaſure an Irregular Field.

THE Way to meaſure irregular Land, is to divide it into Trapeziums and Triangles, thus:

Firſt, view over tne Field, and ſet up Marks at every Angle, and by thoſe Marks you may ſee where to have a Trapezium, as A B C I in the following Figure.



Then begin and measure in a direct Line from A towards C; but when you come to (a), set up your Cross, and try whether you be in a Square to I (as is before directed); and then measure the Perpendicular
Ch. L.
a I, which is 4.82; then measure forward again towards C, but when you come to (b) set up your Cross,

Crofs, and try whether you be in the Place where the Perpendicular will fall; then meafure the Perpendicular b B, which is 2.06 Ch. L.; then continue your Meafure to C, and you will find the whole Diagonal 9.42 Ch. L.

Then proceed to meafure the Trapezium CDHI, beginning at C, and meafuring along the diagonal Line towards A; but when you come at (d), fet up your Crofs, and try if you be in the Place where the Perpendicular will fall: Meafure the Perpendicular d D, which is 1.46 Ch. L., and then meafure forward till you come at (c), and there, with your Crofs, try if you be right in the Place where the Perpendicular will fall, and meafure the Perpendicular cI, which is 3 Chains; and from (c) continue your Meafure to H, and you will find the whole Diagonal 12.36 Ch. L.

Then proceed to meafure the Trapezium HGED, beginning at H, and meafuring along the diagonal Line towards E; but when you come to (f) try with your Crofs, if you be in the Place where the Perpendicular will fall; and meafure the Perpendicular f G, which is 4.48; then continue on your Meafure from (f) till you come to (g), and there try if you be in a Square with the Perpendicular g D; and meafure the faid Perpendicular, which is 2.94 Ch. L.; then meafure on from (g) to E, and you will find the whole Diagonal to be 11. 34 Ch. L.

Then meafure the Triangle EFG, beginning at E, and meafuring along the Bafe EG, till you come at (h), and there with your Crofs try if you be in the Place where the Perpendicular will fall; and meafure the Perpendicular h F, which is 3 14 Ch. L., continue your Meafure to G, and you will find the whole Bafe to be 9.12 Ch. L.; fo you have finifhed your whole Field.

I have been the larger upon the Explanation of this Problem, becauſe moſt Grounds lie in ſuch irregular Forms.

Caſt up the three Trapeziums ſeverally, and alſo the Triangle; and add all the ſeveral Areas together into one Sum, which will be the Area of the whole irregular Plot.

See the Work.

bB=2.06	9.42	See Sect. VI. Chap. I. Part II.
aI=4.82	3.44	
Sum 6.88	3768	
	3768	
Half 3.44	2826	
	3.24048=Area of ABCI.	

dD=1.46	12.36
cI=3.00	2.23
Sum 4.46	3708
	2472
Half 2.23	2472
	2.75628=Area of CIHD.

fG=4.48	11.34
gD=2.94	3.71
Sum 7.42	1134
	7938
Half 3.71	3402
	4.20714=Area of HGED.

Baſe

Base=9.12

See Sect. V. Chap. I. Part II.

Half=4.56
Perpend. 3.14

1824
456
1368

1.43184=Area of the Trian. EFG.
3.24048=Area of ABCI.
2.75628=Area of CIHD.
4.20714=Area of HGED.

Sum 11.63574=Area of the Whole.
4

2.54296
40

21.71840

A. R. P.
Facit 11 2 21

F I N I S.

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